Determination of anisotropic velocity models from walkaway VSP data acquired in the presence of dip

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ABSTRACT
Wide-aperture walkaway vertical seismic profile (VSP) data acquired through transversely isotropic horizontal layers can be used to determine the P phase-slowness surface, local to a receiver array in a borehole. In the presence of dip, errors in the slowness surface may occur if the medium is assumed to be layered horizontally. If the acquisition plane is oriented parallel to the dip direction, the derived slowness is too large for sources offset from the well in the down-dip direction and too small for sources offset from the well in the up-dip direction. For acquisition parallel to the strike of the layers, the recovery of the P phase-slowness in the vicinity of the receiver array is excellent. It is therefore preferable to orient the walkaway VSP in the strike direction to estimate the anisotropic parameters of the medium in the vicinity of a receiver array. However, this may not be possible. If the dip direction of all layers has the same azimuth, the variation of walkaway traveltimes with azimuth has a simple form. This allows data from a single walkaway VSP extending both sides of a well to be inverted for the local anisotropic P phase-slowness surface at the receivers even in the presence of dip. If data are acquired at more than one azimuth, the dip direction can be determined.

INTRODUCTION
Failure to account for anisotropy in seismic processing may lead to errors in velocity analysis, normal moveout (NMO), dip moveout (DMO), migration, time-to-depth conversion, and AVO analysis. To extend seismic processing to anisotropic media, an anisotropic velocity model is required. For horizontally layered media, a direct estimate of the anisotropic phase-slowness surface local to a receiver array in a borehole can be obtained from arrival times measured in a wide-aperture walkaway vertical seismic profile (VSP) experiment (Gaiser, 1990; Miller et al., 1994). This method is illustrated in Figure 1. The components \( p_i \) of the slowness vector \( \mathbf{p} \) at position \( x \) with components \( x_i \) are given by the gradient in the traveltme measured at \( x \):

\[
p_i = \frac{\partial t}{\partial x_i}.
\]  

For a vertical well, the vertical component of the slowness vector at the receiver array for a given source position can be calculated directly from the arrival times of the wavefront measured at the geophones placed within the formation of interest (see Figure 1a). If the medium is layered horizontally, the horizontal component of the slowness vector is constant (Snell's law), independent of depth, and can therefore be calculated using the variation with source position of the arrival time at a fixed receiver (Gaiser, 1990; Miller et al., 1994) (see Figure 1b). If the medium is then assumed to be transversely isotropic with a vertical axis of symmetry, estimates of the density-normalized elastic stiffnesses of the medium local to the receiver array may be made (Miller and Spencer, 1994).

In the presence of dip, the horizontal component of the slowness vector is not conserved and the method of Gaiser (1990) and Miller et al. (1994) may be inaccurate. The purpose of this paper is to examine the effect of dip on the determination of the \( P \) phase-slowness in the vicinity of the receiver array and to propose a solution for the errors which result.

AN ANISOTROPIC LAYERED MODEL
To illustrate the use of equation (1), anisotropic ray tracing was performed for a simulated marine walkaway VSP experiment in a horizontally layered model using the ray-tracing scheme of Costa et al. (1991). The model is shown on the left of Figure 2 and consists of an alternating sand/shale sequence with anisotropic shales. The water depth is 100 m, and shales make up about 75% of the model in agreement with the percentage of shales occurring in many sedimentary basins (Jones and Wang, 1981). The elastic moduli were assumed to be constant.
within each layer, the seismic velocities within the sandstone and shale layers assumed to be given by

\[ v(z, \theta) = v_0(\theta)(1 + gz), \]  

(2)

where \( z \) is the depth from the sea bottom to the middle of the layer, \( \theta \) is the phase angle measured from the vertical, and \( g \)

defines the variation of velocity with \( z \). A value \( g = 5 \times 10^{-4} \text{ m}^{-1} \) was chosen. For this choice of gradient \( 1 + gz = 2 \) when \( z = 2 \times 10^3 \text{ m} \) so that the velocities double on going from the sea bottom to a depth of 2000 m. For the sandstone layers, \( v_0 \) is assumed to be equal to 1600 m/s, independent of direction. For the shales, \( v(z, \theta) \) at \( z = 2000 \text{ m} \) is assumed to be that given by elastic constants derived from those of Greenhorn shale measured in Jones and Wang (1981) as described in the Appendix. The variation in the vertical and horizontal P-wave velocity for each layer in the model is shown in Figure 2.

Two receivers within the bottom shale layer at depths 1790 m and 1810 m were assumed to be placed in a vertical well through the model. Traveltimes \( t(x) \) from sources on the surface at source-receiver horizontal offset \( x \) were computed using a source spacing of 80 m out to a maximum offset of ±4 km. For this case, the computed traveltimes are symmetric about the well. Figure 3a shows the computed \( t^2 \) versus \( x^2 \) curve for the receiver at 1790 m depth. The variation of traveltimes with offset is seen to be nonhyperbolic as a result of the horizontal layering and the anellipticity of the shales (Sayers, 1995).

The P phase-slowness curve computed from the traveltimes using equation (1) is shown in Figure 3b together with the exact result.

**EFFECT OF DIP**

The calculations were repeated for the case when the symmetry axes of the shale layers and all interfaces below the sea floor are tilted downwards by 5° in the positive offset direction. The depths of the layers at 4 km offset from the well in the up-dip direction are assumed to be the same as in Figure 1. Two receivers at 2140 m and 2160 m depth within the lower shale layer were assumed.

**Acquisition plane parallel to the dip direction**

Figure 4a shows the computed P-wave traveltimes versus horizontal source-receiver offset for a receiver at 2140 m depth for acquisition parallel to the dip direction. The corresponding \( t^2 \) versus \( x^2 \) curve is shown in Figure 4b. The maximum horizontal source-receiver offset is ±4 km, and the source spacing is 80 m as before. Figure 5a shows the corresponding P phase-slowness curve computed from equation (1) together with the exact result. For comparison, Figure 5b shows the corresponding P phase-slowness curve for the same model but with the symmetry axis of the shale layers vertical. In agreement with the conclusion of Gaiser (1990), a 5° dip is seen to have a significant effect on the derived slownesses, with the derived slowness being too large for sources offset from the well in the down-dip direction and being too small for sources offset from the well in the up-dip direction. The tilt of the symmetry axis is seen to have a minor effect on the derived slowness surface.

**Acquisition plane parallel to the strike direction**

Figure 6a shows the computed \( t^2 \) versus \( x^2 \) curve for a receiver at 2140 m depth for acquisition parallel to the strike direction. For this case, the traveltimes are symmetric about the well. Figure 6b shows the corresponding P phase-slowness curve computed from equation (1) together with the exact result. It is seen that for an acquisition parallel to the strike, the recovery of the elastic stiffnesses in the vicinity of a receiver
array is excellent, the tilt of the symmetry axis having only a minor effect on the derived slowness surface. To estimate anisotropic parameters of the medium, a walkaway VSP in the strike direction is therefore preferable.

**CORRECTION FOR THE EFFECTS OF DIP**

Consider a walkaway VSP acquired in the presence of dip with source points extending both sides of a surface point \( O \), as shown schematically in Figure 7. Let \( t(x, \phi) \) denote the traveltime measured at a receiver at location \((x_0, y_0, z)\) from a source at horizontal offset \( x \) from \( O \) on a walkaway line at azimuth \( \phi \) measured with respect to a reference set of axes \( X_1X_2X_3 \) at \( O \). Using reciprocity, the measured traveltimes are identical to those that would be measured in an experiment with receivers at the shotpoints and a single source at the receiver position.

Consider a ray beginning at the receiver location and emerging at \( O \). The traveltime \( t(x, \phi) \) at offset \( x \) along the walkaway line is determined by the wavefront emerging within the plane \( P \) defined by the direction of the walkaway and the direction of the emerging ray. In general, the plane \( P \) will not be vertical. Following Hubral and Krey (1980), let \( \gamma_0 \) be the angle between the emerging ray and a line \( L \) perpendicular to the \( x \)-direction and lying within the plane \( P \), defined such that \( \gamma_0 \) is positive if \( L \) falls between the positive \( x \)-direction and the emerging ray direction. If the top layer is isotropic, as is the case for marine walkaway VSP surveys, with \( P \) wave-velocity \( v \), and if \( R_0 \) is the radius of curvature of the emerging wave within the plane \( P \), it follows from Hubral and Krey (1980) that to second order...

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**Fig. 3.** (a) \( t(x)^2 \) versus \( x^2 \) measured at a receiver at depth \( z_r = 1790 \) m in the horizontally layered model shown in Figure 2. (b) Variation in \( P \) phase-slowness (---) with angle computed for the bottom shale layer using equation (1) and the traveltimes obtained by ray tracing for a maximum source-receiver offset of \( \pm 4 \) km. Also shown is the exact result (---) computed using the correct elastic stiffnesses of the layer.

**Fig. 4.** (a) Traveltime \( t(x) \) versus horizontal source-receiver offset \( x \) and, (b) \( t(x)^2 \) versus \( x^2 \) measured for a receiver at depth \( z_r = 2140 \) m. The symmetry axis of the shale layers and all interfaces below the sea floor are tilted by 5° in the positive offset direction, the depths of the layers at 4 km offset from the well in the negative offset direction being the same as in Figure 2.
in the offset, the traveltim \( t(x, \phi) \) at offset \( x \) is related to the traveltim \( t(0) \) at zero offset by:

\[
t(x, \phi) = t(0) + \frac{\sin \gamma_0}{v} x + \frac{\cos^2 \gamma_0}{2vR_0} x^2 + \cdots. \tag{3}
\]

For small dips and \( \theta_0/\gamma_0 \ll 1 \), the offset \( x_{\text{min}} \), corresponding to the minimum traveltime on the walkaway line, will lie in the vicinity of \( O \). Solving equation (3) in the neighborhood of \( O \) gives

\[
x_{\text{min}} = -\frac{R_0 \sin \gamma_0}{\cos^2 \gamma_0}. \tag{4}
\]

For low velocity near-surface layers, \( \gamma_0 \) will be small and the minimum time will occur at \( x_{\text{min}} = -\gamma_0 R_0 \) with \( \gamma_0 \) measured in radians.

Consider now the sum of traveltimes \( t(x, \phi) + t(-x, \phi) \) for two sources at offsets \( x \) and \( -x \). This is invariant under a rotation of \( \pi \) about the \( x_3 \)-axis through \( O \). It is also invariant under a change in the sign of \( x \) and is therefore an even function of \( x \). If \( O \) is chosen to lie directly above the receiver and the anisotropic layers are transversely isotropic with a vertical axis of symmetry (TIV), the result of combining these two operations is equivalent to changing the sign of the dip. \( t(x, \phi) + t(-x, \phi) \) is therefore invariant under a change in sign of the dip. This is true even if the top layer is TIV. Thus, for gently dipping reflectors, \( t(x, \phi) + t(-x, \phi) \) is of second-order in the dip and

**Fig. 5.** Variation in P phase-slowness (\( \cdots \)) with angles computed assuming horizontal layers for the bottom shale layer when (a) the symmetry axis of the shale layers and all interfaces below the sea floor are tilted by \( 5^\circ \) in the positive offset direction, the depths of the layers at 4 km offset from the well in the negative offset direction being the same as in Figure 2. Also shown is the exact result (\( -\)) computed using the correct elastic stiffnesses of the layer. The case when only the interfaces are tilted is shown in (b).

**Fig. 6.** (a) \( t(x)^2 \) versus \( x^2 \) for acquisition parallel to the strike measured at a receiver at depth \( z_r = 2140 \) m when the symmetry axis of the shale layers and all interfaces below the sea floor are tilted by \( 5^\circ \), the depths of the layers at 4 km offset from the well in the up-dip direction being the same as in Figure 2. (b) Variation in P phase-slowness (\( \cdots \)) with angle computed for the bottom shale layer. Also shown is the exact result (\( -\)) computed using the correct elastic stiffnesses of the layer.
An isotropic Velocity Analysis is therefore independent of azimuth to first-order in dip. It follows that for small dips and a walkaway line extending both sides of the receiver location, \( t(x, \phi) + t(-x, \phi) \) may be used to obtain a dip-independent estimate of the anisotropic phase-slowness in the vicinity of the receiver array. This will be illustrated below for the case of a 5° dip, dips in the range 0° to 5° being fairly common. The error increases as the square of the dip. However, for dips greater than 5° the assumption that the velocity does not vary laterally within the layers may be untenable.

To test this approach, ray tracing through the model was also performed at 15°, 30°, 45°, 60°, 75°, 195°, 210°, 225°, 240°, and 255° to the dip direction. Figure 8a shows the variation in traveltime with azimuth for various offsets. Also shown is a fit of the data to the expression

\[
t(x, \phi, \eta) = a(x) + b(x) \cos(\phi - \eta),
\]

where \( x \) is the horizontal source-receiver offset, \( \phi \) is the azimuth of the walkaway, and \( \eta \) is the azimuth of the down-dip direction. It is seen that the variation of walkaway traveltimes with azimuth is well approximated by equation (5). This allows the traveltimes \( t_{\text{strike}}(x) \) in the strike direction to be estimated from a single wide-offset walkaway VSP experiment extending both sides of the well as

\[
t(x, \phi, \eta) + t(x, \phi + \pi, \eta) = 2a = 2t_{\text{strike}}(x).
\]

The traveltimes estimated in the strike direction using equation (6) were input into equation (1) to compute the \( P \) phase-slowness surface. The recovery of the \( P \) phase-slowness surface using this method was found to be excellent for all azimuths considered. As an example, the derived \( P \) phase-slowness surfaces for the 30° azimuth without the dip correction is shown in Figure 9a. Figure 9b shows the slowness surface derived using equation (6) for the same data. The agreement with the exact slowness surface is good. It is convenient to denote by \( a_{ij} \) the density-normalized elastic stiffness moduli in the 6 x 6 condensed notation. Miller and Spencer (1994) show that \( a_{33} \), and the combination \( a_{13} + 2a_{55} \) can be obtained from the \( P \) phase-slowness surface using a simple linear method. Using this approach, the computed \( P \) phase-slownesses shown in Figure 9b yield the values \( a_{11} = 11.4342 \text{ km}^2/\text{s}^2 \), \( a_{33} = 9.2629 \text{ km}^2/\text{s}^2 \), and \( a_{13} + 2a_{55} = 9.1873 \text{ km}^2/\text{s}^2 \), which are in satisfactory agreement with the exact values \( a_{11} = 11.3474 \text{ km}^2/\text{s}^2 \), \( a_{33} = 9.2537 \text{ km}^2/\text{s}^2 \), and \( a_{13} + 2a_{55} = 9.0370 \text{ km}^2/\text{s}^2 \).

**DETERMINATION OF THE DIP DIRECTION**

If data are acquired at more than one azimuth, it may be possible to determine the dip direction. One possible geometry in the marine environment is shown in Figure 10. For this case, let the traveltime at horizontal offset \( x \) in the three linear segments labeled 1, 2, and 3 in Figure 10 be denoted by \( t_1(x) \), \( t_2(x) \), and \( t_3(x) \). It follows from equation (5) that

\[
\begin{align*}
\frac{t(x, \phi, \eta) + t(x, \phi + \pi, \eta)}{2} & = a(x) + b(x) \cos(\phi - \eta) \\
& = a(x) + b(x) \cos(\phi - (\phi + \pi)) \\
& = a(x) + b(x) \cos(\phi - \phi - \pi) \\
& = a(x) + b(x) \cos(-\pi) \\
& = a(x) - b(x) \\
& = 2t_{\text{strike}}(x).
\end{align*}
\]

The traveltimes estimated in the strike direction using equation (6) were input into equation (1) to compute the \( P \) phase-slowness surface. The recovery of the \( P \) phase-slowness surface using this method was found to be excellent for all azimuths considered. As an example, the derived \( P \) phase-slowness surfaces for the 30° azimuth without the dip correction is shown in Figure 9a. Figure 9b shows the slowness surface derived using equation (6) for the same data. The agreement with the exact slowness surface is good. It is convenient to denote by \( a_{ij} \) the density-normalized elastic stiffness moduli in the 6 x 6 condensed notation. Miller and Spencer (1994) show that \( a_{11} \), \( a_{33} \), and the combination \( a_{13} + 2a_{55} \) can be obtained from the \( P \) phase-slowness surface using a simple linear method. Using this approach, the computed \( P \) phase-slownesses shown in Figure 9b yield the values \( a_{11} = 11.4342 \text{ km}^2/\text{s}^2 \), \( a_{33} = 9.2629 \text{ km}^2/\text{s}^2 \), and \( a_{13} + 2a_{55} = 9.1873 \text{ km}^2/\text{s}^2 \), which are in satisfactory agreement with the exact values \( a_{11} = 11.3474 \text{ km}^2/\text{s}^2 \), \( a_{33} = 9.2537 \text{ km}^2/\text{s}^2 \), and \( a_{13} + 2a_{55} = 9.0370 \text{ km}^2/\text{s}^2 \).
\[ t_1(x) = a + b \cos(\phi_1 - \eta), \quad (7) \]
\[ t_2(x) = a + b \cos(\phi_2 - \eta), \quad (8) \]
\[ t_3(x) = a + b \cos(\phi_1 + \pi - \eta), \quad (9) \]

and therefore that
\[ \phi_1 - \eta = \tan^{-1}\left( \frac{\cos \Delta \phi - \frac{(2t_2 - t_1 - t_3)}{(t_1 - t_3)}}{\sin \Delta \phi} \right), \quad (10) \]

where \( \Delta \phi = \phi_2 - \phi_1 \).

Consider, for example, the case \( \phi_1 - \eta = 30^\circ, \phi_2 - \eta = 75^\circ \) so that \( \Delta \phi = 45^\circ \). Figure 11 shows the value of \( \phi_1 - \eta \) computed as a function of horizontal offset for this example. The correct value is \( \phi_1 - \eta = 30^\circ \).

**EFFECT OF ERROR IN THE RECEIVER LOCATION**

In the examples given above, the receivers are assumed to lie directly below \( O \) in Figure 7. In many situations, this will not be the case because of, as an example, the uncertainty in the receiver location. The uncertainty in receiver location may be removed by transforming the origin to the minimum traveltime.

**Fig. 9.** Variation in \( P \) phase-slowness (\( \cdots \)) with angle computed for the bottom shale layer when the symmetry axes of the shale layers and all interfaces below the sea floor are tilted by 5\(^\circ\), the depths of the layers at 4 km offset from the well in the up-dip direction being the same as in Figure 2. The data are acquired at an azimuth of 30\(^\circ\) with respect to the down-dip direction. The slowness calculated assuming horizontal layers is shown in (a) while that computed using equation (6) is shown in (b). Also shown is the exact result (\( -\)) computed using the correct elastic stiffnesses of the layer.

**Fig. 10.** Possible wide-offset marine walkaway VSP acquisition geometry.

**Fig. 11.** Value of \( \phi_1 - \eta \) computed from equation (10) as a function of horizontal source-receiver offset for the case \( \phi_1 - \eta = 30^\circ, \phi_2 - \eta = 75^\circ \) so that \( \Delta \phi = 45^\circ \). The correct value is \( \phi_1 - \eta = 30^\circ \).
The shales in the model are assumed to be transversely isotropic. If the symmetry axis is chosen as the \(x_3\)-direction, the nonzero elastic stiffnesses \(C_{ij}\) are \(C_{11} = C_{22}, C_{33}, C_{44} = C_{55}, C_{66}, C_{13} = C_{23},\) and \(C_{12} = (C_{11} - 2C_{66}).\) It is convenient to write the \(C_{ij}\) in the form

\[
C_{ij} = C_{ij}^0 + \Delta_{ij},
\]

where \(C_{ij}^0\) are the elastic stiffnesses of an isotropic comparison medium, and \(\Delta_{ij}\) is the difference between \(C_{ij}\) and \(C_{ij}^0.\) The isotropic comparison model was chosen as the isotropic medium most similar to the anisotropic medium in the sense of Federov (1968) and has second-order Lamé elastic constants given by

\[
15\lambda = C_{11} + C_{22} + C_{33} + 4C_{12} + 4C_{23} + 4C_{44} - 2C_{44} - 2C_{55} - 2C_{66},
\]

\[
15\mu = C_{11} + C_{22} + C_{33} - C_{12} - C_{23} - C_{13} + 3C_{44} + 3C_{55} + 3C_{66}.
\] (A-3)

The anisotropic velocity of the shales is assumed to be given by equation (2) where \(z\) is the depth from the sea bottom to the middle of the layer. Here, \(v_0(\theta)\) was chosen such that for \(z = 2000\) m, \(v(z, \theta)\) is the velocity given by elastic constants derived from those of Greenhorn shale measured by Jones and Wang (1981) by choosing the \(\Delta_{ij}\) to be half the values calculated for Greenhorn shale using the isotropic comparison medium of Federov.