ABSTRACT

A highly efficient and accurate tool for predicting the seismic response of reservoir fluid flow has been developed which integrates the finite-difference injection method with a reservoir simulator and a petrophysical model. Finite-difference methods allow for the full response to be synthesized as the wavefield interacts with a seismic model. This includes wave propagation in arbitrary heterogeneous anisotropic and anelastic media, scattering, and mode conversions. The finite-difference injection method, in turn, can be used to efficiently synthesize the seismic response from models after local alterations to the model. Thus, it is ideally suited for time-lapse seismic studies.

The modeling methodology is demonstrated on a case study from the Gullfaks field in the North Sea. Six complete marine seismic surveys over the reservoir at different stages during waterflood oil production were synthesized. A total of 180 shot gathers were synthesized with computational savings of a factor of 54 after one single full simulation. The computational savings for the analogous 3-D study are 370 or greater after the initial simulation.

The surface seismic response acquired along a towed streamer was processed through to stack and migrated. In a noise-free environment the replacement of oil by water at a constant pressure caused visible changes in the synthetic seismic response that closely correspond to the impedance changes in the reservoir because of fluid flow.

Downhole permanent sensor or vertical seismic profiling configurations were also considered; they provided a particularly suitable acquisition geometry for time-lapse seismic monitoring. The recorded wavefields during and before production were greatly different (comparable to the magnitude of the wavefield itself). Moreover, multicomponent measurements may allow for elimination of changes attributable to environmental effects in the overburden and source characteristics. The study also indicates that monitoring the phase change of a reflector below a reservoir may provide a fluid flow indicator. The simulation technique thus provides an important tool for designing downhole surveys and deploying permanent sensors.

INTRODUCTION

In time-lapse seismic studies, the difference between two or more seismic data sets acquired at different times during the production process is used to infer changes in the distribution of fluids because of production. To decide whether a repeat seismic survey is able to detect fluid changes predicted by reservoir simulation, and to check if an existing seismic data set is consistent with the reservoir model, a seismic forward model is required. The predicted distributions of water saturation, pressure, and temperature from a reservoir simulator can be combined with rock properties, via a petrophysical model, to provide the elastic moduli for the seismic modeling.

Previous work includes the use of complex reservoir models based on the results of a reservoir simulator, providing the input for a simple convolutional approach to synthesize seismic data. Recent examples of such studies include a Gulf of Mexico field (Huang et al., 1998), the Fulmar field (Johnston et al., 1997), and Magnus field (Watts et al., 1995) in the North...
Sea. These studies construct a detailed petrophysical model that predicts the acoustic impedance from a combination of well-log and laboratory data, and theoretical relationships. The acoustic impedance is then calculated for every block in the reservoir simulator model, and convolution with a wavelet is used to create synthetic seismic data. Packwood (1997) follows a similar methodology but uses a propagator matrix method to synthesize zero-offset synthetics. Biondi et al. (1998) use a first-order Born approach to synthesize data at near and far offsets. Their models consist of an overburden with a linear velocity gradient and a reservoir based on Eclipse (a mark of Schlumberger) simulations. Gjøystdal et al. (1998) use a more accurate hybrid modeling method that combines ray tracing in the overburden and an acoustic finite-difference method in the reservoir area. However, the fluid flow in the reservoir is modeled by perturbations of the parameters in a single homogeneous layer.

In this paper, the properties of the reservoir model and their distribution are determined using a reservoir simulator. We combine the realism of such a mechanism for generating seismic models with the advantages of finite-difference seismic forward modeling. Finite-difference methods inherently include all converted waves, multiples, and diffractions. Moreover, realistic material properties including anisotropy and attenuation can be accounted for. Although accurate, traditional finite-difference calculations can be highly computationally expensive, and their applicability is therefore limited by available computers. We use a finite-difference-based technique referred to as the finite-difference injection method. The technique, developed by Robertsson and Chapman (2000), efficiently computes the seismic response from a seismic model subject to one or several changes within subvolumes. Thus, it is ideally suited for time-lapse studies.

We demonstrate the application of this approach to time-lapse seismic studies with a case study based on the Gullfaks field in the North Sea, where a successful real time-lapse study was carried out (Sønneland et al., 1997). Several marine seismic surveys are simulated over the model during production. Surface seismic data are then processed through to stack and migrated. The response from downhole measurements [vertical seismic profiling (VSP) or permanent sensors] are also analyzed. The simulations demonstrate that the combination of highly cost-effective finite-difference simulations with realistic reservoir modeling provides a valuable tool for understanding time-lapse seismic data and planning acquisition.

**FINITE-DIFFERENCE INJECTION**

A detailed description of the finite-difference injection technique is found in Robertsson and Chapman (2000). The technique is based entirely on finite differences, which ensures a highly accurate solution that fully accounts for the interaction of the wavefield with the seismic model. The basic idea behind the method is that once a finite-difference response on a full finite-difference mesh has been calculated, the altered response resulting from local changes in the seismic model can be updated by recomputing only the response on a finite-difference submesh. This reduces the computational cost significantly in terms of the number of calculations and memory for storage of material parameters and variable fields. Regions where models are altered and in whose neighborhood finite-difference solutions are recalculated are referred to as injection subvolumes. The term injecting a wavefield refers to introducing a wavefield recorded in a previous finite-difference simulation along the closed injection surface defining the injection subvolume (this becomes the source field that drives the finite-difference simulation).

The finite-difference injection technique is based on (1) the continuity of wavefields and (2) the superposition principle. Figure 1 illustrates the volumes and surfaces of a model that are fundamental to the technique. Different solutions are connected in different regions of the model by using the superposition principle and ensuring continuity of the superimposed wavefields. The surface \( S_e \) encloses the altered region of the model, the injection subvolume \( V_i \). Exterior to this surface is volume \( V_e \) (theoretically, \( V_e \) need not be connected, but in practice it normally is). Surrounding \( V_i \) is the boundary \( S \) of the finite-difference submesh used to calculate the altered solution. This boundary limits the size of the finite-difference mesh used in the calculations. Its only purpose is to act as an absorbing boundary condition (just as the boundary of the complete finite-difference mesh acts as an absorbing boundary). It must be larger than \( S_e \) so that the boundary conditions do not interfere with the finite-difference solution within \( V_i \). We denote the part of \( V_i \) inside \( S \) as \( V^1_i \) and the part outside as \( V^2_i \), i.e.,

![Figure 1](https://example.com/fig1.png)

**FIG. 1**. The injection subvolume \( V_i \) (the reservoir) is enclosed by the surface \( S_e \). The model in the exterior volume \( V_e = V^1_i \cup V^2_i \) remains unchanged. The absorbing boundary of the finite-difference submesh \( S \) lies in the unaftered volume \( V^2_i \). A surface \( S^1 = S^1_e \cup S^1_i \) is defined between the model alterations and the receivers (triangles). The surface integral is restricted to \( S^1 \) within the submesh. The source (star) is outside the finite-difference submesh.
In anisotropic media (Robertsson and Chapman, 2000), the finite-difference submesh is \( V_e' \cup V_i \).

Thus, the finite-difference submesh is \( V_e' \cup V_i \).

In this paper, the receivers are located outside the boundary \( S \), i.e., outside the finite-difference submesh. We therefore construct the surface \( S_b \) intersecting the finite-difference submesh between the injection subvolume \( V_i \) and the receivers (triangles in Figure 1). As is well known (Tan, 1975; Aki and Richards, 1980), Betti’s theorem leads to a representation integral that expresses the solution in terms of a surface integral and a volume integral over enclosed sources, i.e.,

\[
v_i = \int_V G_{ij} f_i \, dV + \int_S (G_{ik} \sigma_{lj} - \Sigma_{ijk} v_ik) n_j \, dS,
\]

where \( f_i \) corresponds to sources within the volume \( V \) and where \( \sigma_{ij} \) and \( v_i \) in the surface integral correspond to stress and velocity along the surface \( S \) with normal \( n_j \) surrounding \( V \). The quantities \( G_{ij} \) and \( \Sigma_{ijk} \) are the velocity and stress Green’s tensors from the observation point (where \( v_i \) on the left-hand side is calculated) to the point of integration. In the time domain, the products are time convolutions; in the frequency domain, they are simple products. The representation integral (2) allows the solution to be propagated from this surface to the receivers. In practice, the surface integral is restricted to the portion of this surface within the finite-difference submesh, \( S_e' \), resulting in possible truncation errors. For completeness, we refer to the part of \( S_e \) outside the finite-difference submesh as \( S_e'' \), i.e.,

\[
S_e = S_e' \cup S_e''.
\]

In summary, one full finite-difference simulation is performed initially using the entire model in \( V \), and the wavefield is recorded along the surface \( S_e \). The recorded wavefield is then injected along \( S_e \), in subsequent finite-difference simulations on the subvolume \( V_e' \cup V_i \). Inside \( V_i \), the wavefield will be that of the new model. If \( V_i \) has not been altered, the injected wavefield cancels perfectly along \( S_e \) so that the wavefield in \( V_i \) is as close to zero as machine precision allows. However, if \( V_i \) has been altered, the wavefield in \( V_i \) will consist of the part of the wavefield that corresponds to the change in \( V_i \).

Finite-difference injection synthesized seismograms closely correspond to the wavefield that would have been obtained by executing the full finite-difference simulation using the altered seismic model. The only events that are not accounted for are those corresponding to an alteration in the model that leave the injection region (inside \( S_e' \)), propagate outside the finite-difference submesh \( V_e' \cup V_i \), and then reverberate back inside the grid to the recording locations along \( S_e'' \). Note, however, that independent of how the inner grid is chosen, the finite-difference injection technique is exact up to propagation times after the time of the first arrival corresponding to the traveltime from the region of change, to the absorbing boundary of the inner grid, and back to the recording locations.

The finite-difference injection technique has been implemented using a staggered stress-velocity formulation of the first-order partial differential equations describing 2-D (Cartesian coordinates \( x \) and \( z \)) viscoelastic wave propagation in anisotropic media (Robertsson and Chapman, 2000).

**Finite-difference models with a 1-D background**

This study was performed on an important class of models consisting of a 1-D background where the properties vary (arbitrarily) only with depth. Within this model, a region of arbitrary shape and structure is located (the reservoir). The finite-difference injection field and the Green’s functions for the back extrapolation to the surface can then be calculated on the 1-D background, provided the entire reservoir is located within the injection surface \( S_e \) (see Figure 3). All changes take place within the reservoir.

For models that vary only with depth, Green’s functions are functions of offset for fixed depth levels of start and end points for the Green’s functions. We can therefore record the wavefield along multiple surfaces \( S_e \), and thereby generate the injection field for several shots with different offsets to the reservoir during one single full finite-difference simulation.

During simulations on the subvolume, the wavefield is recorded along \( S_e \). This wavefield can be extrapolated to the receivers at the top of the model (marked by triangles in Figure 1) by using equation (2). To perform this step, we must know the Green’s functions between the points along the discretized integration surface \( S_e \) and the receiver locations at the surface. Again, if the background medium is one dimensional, all Green’s functions can be simulated simultaneously. All particle velocities and several of the stress components are needed along the integration surface to calculate the pressure at the surface using equation (2). If we use the reciprocity theorem, we can replace pressure receivers by explosive sources and record all wavefield quantities at depth along \( S_e \). This step still maintains the high accuracy of the finite-difference approach as opposed to, for instance, using Green’s functions calculated by asymptotic techniques (as in a hybrid technique). Alternatively, a reflectivity method (Kennett, 1983) could also have been used because the background is one dimensional.

In summary, if sources and receivers at the surface are located at the same depth level, all injection surfaces \( S_e \) and Green’s functions for the back extrapolation can be calculated by performing only one single full finite-difference simulation.

**RESERVOIR MODEL**

The overburden model is a geometrically simple 2-D section with properties based on the Gullfaks field, North Sea. The background consists of eleven flat, horizontal layers. At 1963 m depth, a small 457.5-m-wide and 82-m-deep reservoir block is located. Typical or approximate values for parameters have been measured from Gullfaks log and seismic data or estimated from published data or by using relationships and approximations common in the oil industry. The model is not designed to represent the Gullfaks field but demonstrates the application of the finite-difference injection technique in modeling time-lapse seismic data.

**Background properties**

The model has a layered background with a water column at the top. The depths and compressional velocity of the background layers are similar to those found in the region of the Gullfaks field, and the water depth is 120 m. Figure 2 shows \( P \)-wave velocity, \( S \)-wave velocity, and density profiles for the 1-D background (isotropic). The shaded area shows the depth
location of the reservoir. The shear velocity in the background layers was estimated from the compressional velocity using the mud-rock line relationship of Castagna et al. (1985), and the density was estimated from the compressional velocity using Gardner’s equation (Gardner et al., 1974).

In Figure 3, we show the P-wave velocity in the model with the reservoir before production. Toward the bottom of the model is the reservoir. Before production, it is layered and isotropic. Initially, however, the injection wavefields and all Green’s functions are generated using a model that only varies with depth, as given by the profiles (Figure 2).

We opted to model the response without a free surface (by using an absorbing boundary at the top), even though the methodology allows for free surfaces. In effect, this is equivalent to removing all multiples related to the water surface in marine seismic data. An 80-grid-points-thick Kosloff-type absorbing region (Kosloff and Kosloff, 1986) was used as an absorbing boundary. Some additional space at the top of the model means the upper edge of the grid is actually located at \( z = -180 \) m (Figure 3). Typical source and receiver depths for marine seismic surveys \( z = 6 \) m were used.

**Reservoir properties**

The reservoir in this example is based on the properties of the Brent Group reservoirs in the Gullfaks field. A box-shaped, flat-lying (1-D) reservoir was constructed from the log data recorded in well 34/10–34. This vertical well was chosen because it intersected a reasonably complete section of Brent Group reservoir, namely the Tarbert and Ness Formations (Sonneland et al., 1997).

The Brent Group is a thick sequence of shallow-marine deltaic to coastal plain sediments—mostly interbedded sandstones, shales, and coals (Olaussen et al., 1992). The Tarbert Formation is comprised mostly of sandstones with variable porosity and permeability, and the model used in this study covered the 81-m-thick Tarbert Formation only (Figure 4).

The porosity was upscaled from the original 0.152-m sampling to 1.8-m layers by taking the mean value in each 1.8-m layer. The permeability was estimated from an empirical relationship between clay content and porosity derived from North Sea reservoirs (Owen, 1993) and upscaled by means of harmonic averaging. These 1-D porosity and permeability values were used in the box-shaped model, which was input into the Eclipse reservoir simulator as a 15 × 1 × 45 block model. Eclipse inherently models the fluid flow through a reservoir in three dimensions. In this study, we use a 2-D technique for simulating seismic wave propagation. A pseudo-2-D model was therefore constructed in Eclipse with only one grid block in the third dimension. Each grid block was 30.5 m wide, 30.5 m thick, and 1.8 m deep. There was one water injector and one oil producer, and the field was kept at an almost constant pressure during oil production. The properties of the oil were typical of a Brent Group live oil: the oil was 34 API and had a gas–oil ratio of 200 l/l, with a velocity of 900 m/s and a density of 800 kg/m³ at reservoir conditions. The original oil–water contact was at 2003 m, giving a 40-m oil column.

A petrophysical model is needed to predict bulk and shear moduli from rock parameters such as porosity and clay volume and the fluid bulk modulus (calculated from the water saturation predicted by the reservoir simulator). We used the semiempirical velocity-porosity-clay model of Goldberg and Gurevich (1998), although many other petrophysical models are available [e.g., Mavko et al. (1998) and Packwood (1997)]. The inputs to this model were porosity and gamma-ray logs, sampled every 0.152 m. The gamma-ray log was used to calculate the volume fraction of clay. The Biot–Gassman fluid replacement method was then used to combine the frame moduli (at 0.152 m sampling) with the fluid bulk modulus predicted from the output of the reservoir simulator. This gave the bulk and shear moduli at 0.152-m scale. These were then upscaled to 1.8-m-thick layers (for the seismic forward model) using Backus upscaling (Backus, 1962).

Backus upscaling enables a stack of thin, 1-D isotropic [or transversely isotropic (TI)] layers to be replaced by thicker equivalent TI layers (Schoenberg and Muir, 1989). As a rule of thumb, the seismic response is the same as that of the original stack of thin isotropic or TI layers if the upscaled layers are each
thinner than one-tenth of the seismic wavelength. Backus averaging requires a 1-D structure for upscaling. We anticipated that the reservoir cells (30.5 × 30.5 × 1.8 m) were sufficiently close to a 1-D approximation for the method to be valid. We know that the scale on which the pore fluids are mixed affects the seismic velocity of the fluid (Mavko and Mukerji, 1998). In this study we assumed that the oil and water phases were freely and uniformly mixed within each grid cell.

In summary, we have defined porosity and permeability in one dimension, have made a 2-D distribution of fluids via the reservoir simulator, and have transformed this into a 2-D distribution of elastic moduli via a petrophysical model. Figures 5 and 6 show the elastic properties of the reservoir as calculated by this method. The vertical injection well is located along the left edge of the reservoir; the vertical producing well is located along the right edge. Figure 5 shows the elastic modulus $c_{33}$ for six selected time steps during the production. Each time step is one year, so time step 0 corresponds to the reservoir before production starts. Time steps 2, 3, 4, and 5 correspond to production when the waterflood is sweeping the reservoir, and time step 10 corresponds to a late stage in the production when nearly all of the oil has been swept. The waterflood moves from left to right. Figure 6 shows the reservoir at time step 3 and the effect of the fluid replacement on various elastic moduli. At this time step the waterflood has reached the middle of the reservoir. The elastic stiffness $c_{55}$ has not been affected by the fluid changes as expected. Density and $c_{33}$ show moderate changes, and $c_{13}$ shows a fairly large fluid effect.

Absorbing boundaries were used along the sides of the reservoir models in the finite-difference calculations. For the small simulations encompassing the reservoir, 72-m-thick absorbing boundaries of the Kosloff type (Kosloff and Kosloff, 1986) were used.

**TIME-LAPSE SEISMICS STUDY**

**Validation of the finite-difference injection method**

By computing two full finite-difference simulations, we can obtain a reference solution which should correspond closely to the finite-difference injection generated response. We need the response from one simulation using the model in Figure 3 and from one simulation using only the background (i.e., without the small rectangular reservoir). The difference between the responses of these two simulations should closely match the finite-difference injection response for that particular reservoir.

In this test the source (explosive point source) is located at $x = 3594.6$ m and $z = 6$ m. The receivers (recording pressure) are located at the same depth level between $x = 3319.2$ m and $x = 3913.2$ m.

The finite-difference injection solution is computed by first performing one full simulation using the model in Figure 3 without the reservoir. An impulsive source is used so that the Green’s functions between the source level and the surface on top of the reservoir $S'_e$ can be computed at the same time as the injection wavefield. After completion of the full finite-difference calculation, a simulation on the small subgrid (inside $S$) only encompassing the reservoir in Figure 3 and the reflector below it is performed. The upper corner of this subgrid is $(x_0, z_0) = (3004.2$ m, 2050.2 m), and the lower corner is $(x_1, z_1) = (4168.8$ m, 2408.4 m). The subgrid is outlined as a black box in Figure 3. This is the finite-difference injection step.
when the wavefield from the first simulation is injected along the surface $S_e$ around the reservoir. The scattered wavefield is recorded above the reservoir along the same surface $S_e$ as the Green’s functions were recorded in the first simulation. Finally, after completion of the small finite-difference calculation, the scattered wavefield is extrapolated to the surface by means of the representation theorem and convolved with the source wavelet (a 30-Hz Ricker wavelet).

Figure 7 shows the resulting comparison of the finite-difference injection (solid seismograms) result to the full finite-difference simulations (dashed seismograms). Overall, there is an excellent match between the seismograms, even though minor differences are visible. The strongest difference is the event at around 2.18 s that is visible in the finite-difference injection data but not in the full finite-difference response. There are three possibilities for explaining this event:

1) it could be an edge diffraction originating from the back extrapolation using the representation theorem (only performing the integration over $S_e$ as opposed to $S_e' \cup S_e$);
2) it could be caused by an event that has changed after altering the model that propagates outside the finite-difference injection region, leaves the subgrid, and then propagates back inside the subgrid and is recorded; or
3) it is caused by a boundary reflection from the absorbing boundaries in the subgrid.

The first explanation can be ruled out since the arrival time of such a diffraction should be close in time to the first arrival. The second explanation cannot be completely ruled out, even though we expect such events to be very weak in amplitude. Moreover, it would have to be an upgoing event to be extrapolated to the surface. The arrival time of the event coincides with the anticipated reflection of the direct $P$-wave from the lower absorbing boundary in the subgrid. This illustrates that optimal use of finite-difference injection critically depends on efficient absorbing boundary conditions since subgrids are often small and have extreme geometries (e.g., flat and elongated grids encompassing specific targets).

Figure 8 shows another time window from the same data comparison multiplied by a factor of 15. Again, there is an excellent match between the full finite-difference response and the finite-difference injection simulated data, also far into the wavetrain and coda following the first arrival. We conclude that the finite-difference injection algorithm is ideal for simulating time-lapse seismics studies.
Marine seismic survey

The model in Figure 3 was the basis for the simulations of six marine seismic surveys. The reservoir simulation provided models from six different stages during production (see Figure 5). Figure 3 shows the full model with the reservoir for time step 0 (before production) in place. This was replaced by the reservoir corresponding to the respective time steps.

At each reservoir time step, a 30-shot survey was simulated. The shot spacing was 100.8 m, with the first shot location at \( x = 4098.6 \) m and \( z = 6 \) m and the last at \( x = 1175.4 \) m and \( z = 6 \) m. The source consisted of a 30-Hz Ricker wavelet explosive source. The streamer consisted of 322 hydrophones spaced evenly at 12.6 m from the source location and with increasing \( x \) at \( z = 6 \) m.

In summary, a total of 180 shots were simulated using the finite-difference injection technique at a cost of one full finite-difference simulation and 180 finite-difference simulations on the subgrids consisting only of the reservoir and the immediate surrounding. Figure 9 shows a data cube containing the prestack seismic data generated for time step 5.

For each survey at the respective reservoir time steps, we obtained a maximum fold of 83 with a common midpoint (CMP) bin size of 25.2 m, representative of conventional marine seismic streamer acquisition. After sorting the recorded traces into CMPs, the data were NMO corrected using the background P-wave velocity model in Figure 2, assuming hyperbolic move-out. We then stacked the data (the near-offset stack is shown here) and finally migrated it. The 1-D velocity profile of the background was used as the migration velocity model. We would expect to see migration artifacts in the region around the reservoir. However, the simple migration used was sufficient for our purposes since migrating the synthetic data was not the focus of this paper.

Figure 10 shows unmigrated and migrated stacked sections of the reservoir before production. The flat reflectors in the overburden are clearly visible. The more finely layered reservoir is also clearly imaged. Below the reservoir and the reflector beneath it, we see a ghost reflector caused by a multiple between the reservoir and the reflector.

In Figure 11 we show migrated images of the data acquired at the different time steps. We can clearly see the fluid front as it advances through the reservoir (the thin event that changes polarity directly below the reflector at 1.9 s). The diffractions at the edges are caused by the incorrect velocity model in the reservoir used in the migration. This is also why the lowermost reflector below the reservoir at time step 0 (Figure 10) appears to be slightly pulled up.

Figure 12 shows migrated images of differences during and before production. Again, the effects of production as the fluid front advances through the reservoir are clear. The response of the reflector below the reservoir also changes significantly because of the difference in transmitted signal through the reservoir, caused by water replacing oil in the pore spaces. Also of interest is the horizontal event caused by a difference in the fluid at the top of the reservoir.
Modeling Fluid Flow Monitoring

seismic signal in the part of the reservoir that has not been swept of oil during the early time steps (the right-hand side of the reservoir). This is caused by early water breakthrough via a thin, high-permeability layer that lies just below a thin impermeable layer at 2000 m near the base of the oil column.

Figure 13 shows the percent change in acoustic impedance of the reservoir between time step 0 (before production) and time step 2 when the early water breakthrough has already occurred. The figure was obtained from $c_{33}$ and density output from the petrophysical model. The modeled change in acoustic impedance in the area of the main waterflood is between 5% and 10%. The decrease in acoustic impedance in the layer with the early water breakthrough is visible below the impermeable streak.

**Downhole measurements**

Measurements of the wavefield in a borehole close to a reservoir may provide a means of resolving changes in the reservoir because of production that is better than conventional surface seismic data. Both VSP and permanent sensors have been suggested for measurements. Potential limiting factors include the restriction of the measurements to the geometry

![Figure 11](image1.png)  
**FIG. 11.** Stacked migrated data near offset sections from simulated marine seismic surveys: (a) time step 2, (b) time step 3, (c) time step 4, (d) time step 5, and (e) time step 10.

![Figure 12](image2.png)  
**FIG. 12.** Stacked, migrated, near-offset sections of the difference between simulated marine seismic surveys at various time steps and time step 0: (a) time step 2–time step 0, (b) time step 3–time step 0, (c) time step 4–time step 0, (d) time step 5–time step 0, (e) time step 10–time step 0.

![Figure 13](image3.png)  
**FIG. 13.** Difference in acoustic impedance between time steps 0 and 2, normalized to the maximum impedance in the model (%).
of the borehole and the low number of elements that can be deployed in the borehole. Advantages include higher S/N ratio, higher frequency content in the signal, and higher resolution resulting from proximity to the reservoir.

Our technique is also suitable for downhole simulation scenarios. In this section we analyze seismograms recorded along a horizontal line right above the reservoir at $z = 1951.2$ m, simulating a nearby horizontal well. The horizontal recording line runs from $x = 3078$ m to $x = 4093.2$ m. Both horizontal and vertical particle velocities were recorded.

We consider differences in the recorded response between time step 2 (during production) and time step 0 (before production). The principal events in the recorded signal will therefore be upgoing, originating from changes within the reservoir. We investigate arrivals associated with the direct $P$-wave. Other events include a strong $S$-event that is excited as the source pulse interacts with the ocean bottom at the top of the model, and multiples and mode conversions throughout the overburden. However, the direct $P$-wave is by far the strongest wave that causes energy to be reverberated from the reservoir.

Overall, shot gathers display very strong variations with shot location. Figures 14 and 15 show differences in horizontal and vertical components of downhole shot gathers for an explosive source located at $x = 1377$ m and $z = 6$ m. This corresponds to a shot location that is offset horizontally to the left of the reservoir. The intensity of the arrivals in the difference in seismograms is similar in strength to that of the seismograms at the respective time steps. There is a strong arrival from the reservoir with complex first breaks caused by the heterogeneity of the reservoir, but it is difficult to determine much about its actual structure based on these shot gathers.

Figures 16 and 17 show horizontal and vertical component downhole shot gathers for an explosive source located at $x = 3897$ m and $z = 6$ m. This corresponds to the source being

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**Fig. 14.** Difference in horizontal component between time step 2 and time step 0 at shot location $x = 1377$ m. Scaling factor is 1.

**Fig. 15.** Difference in vertical component between time step 2 and time step 0 at shot location $x = 1377$ m. Scaling factor is 1.

**Fig. 16.** Difference in horizontal component between time step 2 and time step 0 at shot location $x = 3897$ m. Event D corresponds to scattering off the fluid front. Scaling factor is 3.

**Fig. 17.** Difference in vertical component between time step 2 and time step 0 at shot location $x = 3897$ m. Scaling factor is 3. A—Top of flooded part of reservoir; B—reflection from interface below reservoir; C—flooded highly permeable layer; D—scattered event off the fluid front.
located roughly right above the reservoir. These sections have been scaled by a factor of three compared to those in Figures 14 and 15. The reflections off the reservoir are weaker because of the smaller incidence angles on the reservoir. On the other hand, this provides deeper penetration, and we see much more structure in the reflections.

The first strong arrival in the left part of section (A) corresponds to the top of the flooded part of the reservoir. This is followed by a wavetrain attributable to scattering and multiple reflections within the reservoir. In the right part of the section, the first strong arrival (C) corresponds to the flooded highly permeable layer. Following the initial wavetrain is another strong arrival (B) accompanied by a wavetrain in the difference plots. This corresponds to energy that propagates through the reservoir and reflects off the interface below it at $z = 2146.5$ m. This is not visible in the right part of the section where the energy propagates through the part of the reservoir that has not been flooded and therefore does not cause a phase shift in the transmitted waves. Such arrivals could therefore prove to be important in determining properties of fluid distributions in certain reservoirs. For example, the phase shift could indicate the position of a gas contact below a horizontal well. Finally, event $D$, which is predominantly horizontally polarized, originates from scattering off the water front in the reservoir.

This example highlights some advantages of acquiring seismic data in the vicinity of the target zone. The seismograms in Figures 16 and 17 resemble the characteristics of the corresponding stacked differential surface seismic images (Figure 12a). This follows from the fact that the source is located roughly right above the reservoir but at a large distance away, so the incoming energy roughly corresponds to a vertical plane wave. The recordings are made right on top of the reservoir, which preserves a clear image. The proposed recording geometry may be particularly suitable for time-lapse seismic monitoring. By separating the incident wavefield from the part of the wavefield that is reflected off the reservoir, it should be possible to eliminate source effects and environmental changes in the near surface and overburden. If multicomponent measurements of the wavefield are made, an up/down separation technique similar to the one described by Amundsen et al. (2000) for demultiple of seabed seismic data could be used.

### COMPUTATIONAL COST AND ACCURACY

The finite-difference injection method yields exact (as far as finite-difference methods go) results after model alterations, independent of the nature of the local change. The only restriction is that the smaller finite-difference grid, where the recalculation takes place, should include the parts of the model that will reverberate scattered/reflected energy caused by the model alteration back to the recording points.

The nature of the original full model is also completely general and is not restricted to 1-D backgrounds. In this paper we chose a 1-D horizontally layered background since such a model allows for highly efficient calculation of injection wavefields and Green’s functions for multiple shots and receivers. (Green’s functions are only a function of offset for given depth locations and not of the exact location.) In the general laterally and vertically arbitrary heterogeneous case, the number of full simulations is proportional to the number of source and/or receiver locations, as opposed to one single full simulation for the special case that we investigated (Robertsson and Chapman, 2000).

A total of 180 shot gathers for various shot locations and reservoir models were required for our time-lapse seismic study. Using the finite-difference injection technique, the computational requirements were significantly brought down to one full simulation and 180 simulations on subgrids only encompassing the reservoir. The subgrid was 46 times smaller than the full model. Since the injection field is zero before the first arrival at the reservoir, additional savings are possible. In our simulations we opted to synthesize the total response well beyond the first arrivals. Hence, savings in terms of the length of simulation time were not as substantial as if we had restricted ourselves to a window following the main $P$-wave arrival. Nevertheless, the small simulations required 54.3 times fewer computations than the full simulations. The analogous 3-D time-lapse study would have enabled computational savings on the order of 370 times. The computational savings could have been even greater with more efficient absorbing boundary conditions. The reservoir models had 72-m-thick absorbing boundary conditions of a sponge type applied on the sides (Kosloff and Kosloff, 1986). If we disregard the absorbing boundary conditions, the subgrid is 75 times smaller than the full model. Hence, more efficient absorbing boundary conditions that are capable of absorbing energy incident over a wide range of angles are needed for optimal use of the finite-difference injection method.

The finite-difference injection technique requires some fairly large files to be stored on disk. For our simulations, all 30 injection fields and Green’s functions for back extrapolation required 15–20 Gbytes of disk space. However, the size of these files could be significantly reduced by using a data compression algorithm.

### DISCUSSION AND CONCLUSIONS

We have illustrated an efficient simulation method for time-lapse seismic studies. The flow of fluids through a reservoir is simulated using a reservoir simulator. The output from the reservoir simulator is converted to a seismic model using a petrophysical model, upscaling from the log scale to a more appropriate seismic scale. The finite-difference injection technique (Robertsson and Chapman, 2000) enables highly efficient and accurate recalculation of seismograms after model alterations.

The finite-difference injection technique was validated against full finite-difference simulations and found to match the full finite-difference generated data excellently. Minor numerical artifacts in the finite-difference injection generated data were caused by inefficiency of the absorbing boundary conditions. Nevertheless, we conclude that the finite-difference injection algorithm is ideal for simulating time-lapse seismic studies.

The seismic response of the reservoir model, including seismic attributes from both the prestack and stacked data, were examined. We showed that in this case study the replacement of oil by water at a constant pressure causes visible changes in the synthetic seismic response that closely correspond to the impedance changes in the reservoir because of fluid flow. However, the focus of this study was not to investigate whether...
certain changes in lithology can or cannot be imaged in a real time-lapse seismic experiment where various sources of noise may obscure the time-lapse response. If a time-lapse seismic survey has been acquired over a field, a model such as this may help determine whether the reservoir model is consistent with the results of the time-lapse survey. Once a set of injection wavefields and Green’s functions has been set up for a reservoir, the seismic response can be modeled for various possible production scenarios—for example, formation of gas caps resulting from a pressure drop. The methodology can then be used to determine if a repeat seismic survey will detect changes as predicted by reservoir simulation and the optimum time to acquire the repeat survey for best seismic imaging conditions.

The proposed simulation method is advantageous in modeling downhole measurements. Such measurements by means of VSPs or permanent sensors may be particularly suitable for time-lapse seismic monitoring. We found the difference of the wavefield recorded during and before production to be quite strong (compared to the strength of the wavefield itself). If the wavefield is separated so that the incoming source signal can be isolated from the response from the reservoir (e.g., Amundsen et al., 2000), it should be possible to remove effects attributable to environmental changes in the overburden as well as source characteristics. Moreover, the incident wavefield roughly corresponds to a plane wave because of the large distance between source and reservoir and the sensors. The proximity of the recording location to the reservoir preserves clear imaging of the target. This can provide an image that closely resembles much of the characteristics in stacked and migrated surface seismic data. We found the recorded section on top of the reservoir to be strongly dependent on shot location. The finite-difference injection technique allowed us to find the best source locations to achieve an image of the reservoir.

Another possibility is to use reflections below the reservoir to monitor fluid flow. The transmitted wavefield undergoes phase shifts because of changes in velocity caused by the replacement of water by oil. In our study, a reflector below the reservoir caused an event that had undergone two-way transmission through the reservoir. The reflector changed visibly under the part of the reservoir that had been swept of oil.

Realistic acquisition effects such as noise may be added to the data, and their effects on the time-lapse signal can be assessed. The effects on the seismic signal of various processing algorithms such as multiple attenuation may also be examined, given that it is possible to model a realistic overburden causing phenomena common in real seismic data such as multiples and mode conversions. Biondi et al. (1998) point out the need to be able to model the seismic response of multiple geostatistical realizations of the reservoir and to test uncertainties attributable to nonuniqueness in the petrophysical model. The use of finite-difference injection as described here will allow fast seismic imaging of multiple geological, petrophysical, and production scenarios with a fine-scale reservoir model. This should in turn enable the design of more reliable and cost-effective time-lapse surveys.

ACKNOWLEDGMENTS

We thank Boris Gurevich and Dave Nichols for many discussions and helpful suggestions. We also thank Statoil and partners for releasing Gullfaks data to the public domain as the Rescue package.

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