Seismic traveltime analysis for azimuthally anisotropic media: Theory and experiment

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ABSTRACT

Natural fractures in reservoirs, and in the caprock overlying the reservoir, play an important role in determining fluid flow during production. The density and orientation of sets of fractures is therefore of great interest. Rocks possessing an anisotropic fabric and a preferred orientation of fractures display both polar and azimuthal anisotropy. Sedimentary rocks containing several sets of vertical fractures may be approximated as having monoclinic symmetry with symmetry plane parallel to the layers if, in the absence of fractures, the rock is transversely isotropic with symmetry axis perpendicular to the bedding plane. A nonhyperbolic traveltime equation, which can be used in the presence of azimuthally anisotropic layered media, can be obtained from an expansion of the inverse-squared ray velocity in spherical harmonics. For a single set of aligned fractures, application of this equation to traveltime data acquired at a sufficient number of azimuths allows the strike of the fractures to be estimated. Analysis of the traveltimes measured in a physical model simulation of a reverse vertical seismic profile in an azimuthally anisotropic medium shows the medium to be orthorhombic with principal axes in agreement with those given by an independent shear-wave experiment. In contrast to previous work, no knowledge of the orientation of the symmetry planes is required. The method is therefore applicable to P-wave data collected at multiple azimuths using multiple offset vertical seismic profiling (VSP) techniques.

INTRODUCTION

Sedimentary rocks frequently possess an anisotropic structure resulting, for example, from fine scale layering, the presence of oriented microcracks or fractures, or the preferred orientation of nonspherical grains or anisotropic minerals. Failure to account for anisotropy in seismic processing may lead to errors in velocity analysis, normal moveout (NMO), dip moveout (DMO), time migration, time-to-depth conversion and amplitude variation with offset (AVO) analysis. Many sedimentary rocks may be described, to a good approximation, as being transversely isotropic (TI) with symmetry axis oriented perpendicular to the bedding plane. Although reflection traveltimes have been extensively studied for TI media (Hake et al., 1984; Byun et al., 1989; Byun and Corrigan, 1990; Delinger et al., 1993; Sena, 1989; Tsvankin and Thomsen, 1994; Sayers, 1995), many formations contain fractures with orientations determined by the stress history of the rock rather than by the orientation of the bedding plane. Natural fractures in reservoirs, and in the caprock overlying the reservoir, play an important role in determining fluid flow during production, and hence the density and orientation of fractures is of great interest (Reiss, 1980; Nelson, 1985). Any fractures open at depth tend to be oriented normal to the direction of the minimum in-situ stress and may therefore lead to significant permeability anisotropy in the reservoir (Sayers, 1990). Since oriented sets of fractures also lead to anisotropic seismic wave velocities, the use of seismic anisotropy to determine the orientation of fractures has received much attention.

In the general case of a rock possessing both an anisotropic fabric and a preferred orientation of vertical fractures, the rock will display an azimuthal anisotropy. For such rocks, observations of the seismic anisotropy have the potential of providing the orientation of the least compressive in-situ stress direction. Although shear waves are considered to be more reliable indicators of fracture orientation than P-waves, considerable interest remains in the use of P-waves for determining fracture orientation, since P-wave acquisition forms the basis of most commercial seismic surveys (Lefevre, 1994; Lynn et al., 1994). Garotta (1989) found that far-offset traveltimes for P-wave data acquired parallel to the dominant fracture azimuth in the Silo Field were less than far-offset traveltimes for acquisition normal to the fractures. Chang and Gardner (1992) suggested that the fracture orientation of a subsurface fracture zone may

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be determined by analyzing P-wave interval velocities. Paul (1993) attributed anomalously low stacking velocities observed on the flanks of the Nan Yi Shan anticline to the presence of localized fracturing. Lynn et al. (1995) found that north-south stacking velocities in a gas field in the Wind River Basin in central Wyoming were slower than east-west stacking velocities, consistent with the presence of east-west trending fractures.

In this paper, a nonhyperbolic traveltime equation is presented that can be used for velocity analysis in the presence of azimuthally anisotropic layered media. The use of this equation is illustrated using ray-traced traveltimes for a fractured shale and measured traveltimes for an azimuthally anisotropic physical model. Tsvankin and Thomsen (1994) have shown that the inversion of surface seismic data for depth-dependent physical model. Tsvankin and Thomsen (1994) have shown that the inversion of surface seismic data for depth-dependent anisotropic velocity models suffers from nonuniqueness because of the trade off between vertical velocity and depth and because of the limited range of offsets normally used. However, under favorable circumstances, borehole seismic data can provide the information required to characterize anisotropic media completely (Miller et al., 1994; Leaney et al., 1996). For this reason the focus in this paper is on the use of multi-offset/multi-azimuth VSP surveys. However, for horizontal layers having a horizontal plane of mirror symmetry, the reflection points lie within the vertical plane containing the source and receiver. The nonhyperbolic traveltime equation presented may therefore be used for analyzing surface reflection times in such media, given a sufficient range of offsets.

SEISMIC TRAVELTIMES IN AZIMUTHALLY ANISOTROPIC MEDIA

Consider the ray geometry for a source-receiver pair in an azimuthally anisotropic homogeneous medium shown in Figure 1. It is convenient to choose a reference set of axes $Ox_1x_2x_3$ with $Ox_3$ vertically down as illustrated in the figure. The square of the traveltime for a wave to travel from the source $S$ to the receiver $R$ in Figure 1 is given by

$$r^2(x) = \frac{z^2}{V^2(\theta, \phi) \cos^2(\theta)},$$

where $r(x)$ is the traveltime at horizontal offset, $x$ and $z$ is the vertical distance traveled. Since the medium is assumed to be azimuthally anisotropic, the ray velocity $V(\theta, \phi)$ is a function of both $\theta$ and $\phi$.

![Fig. 1. Ray geometry in an azimuthally anisotropic homogeneous medium.](image)

It is convenient to introduce a function $r(\theta, \phi)$ defined by

$$r(\theta, \phi) = 1/V^2(\theta, \phi).$$

Here, $r(\theta, \phi)$ may be expanded as a linear combination of spherical harmonics with expansion coefficients $R_{lm}$ (Sayers, 1988; Sayers, 1995) as

$$r(\xi, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} R_{\ell m} P_\ell^m(\xi) \exp(-im\phi),$$

where $\xi = \cos \theta$, $P_\ell^m(\xi)$ is the associated Legendre function and $P_\ell^m(\xi) = (-1)^m P_\ell^m(\xi)$ with $m = -m$.

Since the P-wave ray velocities are centrosymmetric,

$$r(\xi, \phi) = r(-\xi, \phi + \pi).$$

Expanding both sides of this equation in spherical harmonics and using the relation $P_\ell^m(-\xi) = (-1)^m P_\ell^m(\xi)$ gives $R_{\ell m} = (-1)^m R_{\ell m}$. This requires $R_{\ell m}$ to be identically zero when $\ell$ is odd.

It is convenient to write $R_{\ell m}$ in the form

$$R_{\ell m} = \alpha_{\ell m} + i\beta_{\ell m},$$

where the ray velocity $V(\theta, \phi)$ is a real quantity. The following relationships hold: $\alpha_{\ell m} = \alpha_{\ell m}$, $\beta_{\ell m} = -\beta_{\ell m}$ where $m = -m$. Hence, $\beta_{\ell m} = 0$ for $m = 0$. Equation (3) may therefore be written in the form

$$r(\xi, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} P_\ell^m(\xi)[\alpha_{\ell m} \cos m\phi + \beta_{\ell m} \sin m\phi].$$

The traveltime equation for arbitrary anisotropy is then obtained by substituting equation (6) into equation (1). Equation (6) simplifies in the presence of material symmetry. In the following, attention is restricted to the case of layers having monoclinic symmetry. A material with monoclinic symmetry has a plane of mirror symmetry. A sedimentary rock containing several sets of fractures with normals lying in the bedding plane is an example of a medium with a single plane of mirror symmetry if, in the absence of fractures, the rock is transversely isotropic with symmetry axis perpendicular to the bedding plane. If $Ox_3$ is normal to the mirror plane, it follows that $r(\xi, \phi) = r(-\xi, \phi)$ and that $R_{\ell m} = 0$ if $m$ is odd. Equation (6) then reduces to the form

$$r(\xi, \phi) = \alpha_{00} P_0^0(\xi) + \alpha_{20} P_2^0(\xi) + 2(\alpha_{42} \cos 2\phi + \beta_{22} \sin 2\phi) P_2^2(\xi) + 2(\alpha_{42} \cos 2\phi + \beta_{22} \sin 2\phi) P_4^2(\xi) + 2(\alpha_{44} \cos 4\phi + \beta_{44} \sin 4\phi) P_4^4(\xi).$$

Note that in deriving this equation, it is not assumed that the anisotropy is weak, the form of the equation following from the symmetry of the medium. Although no terms of order higher than $\ell = 4$ are included, equation (7) gives a good representation even for relatively large anisotropy, as will be shown below. The main reason for this is that the elastic stiffness tensor is of fourth rank. For realistic data errors and the often rather limited aperture of seismic acquisition the inclusion of terms of order higher than $\ell = 4$ is difficult to justify.
Equation (7) may be written in the form

\[ r(\xi, \phi) = A_0 + A_1 \cos^2 \theta + A_2 \cos^4 \theta, \]  

where \( \xi = \cos \theta \). This equation is of the same form as that derived by Byun et al. (1989) for a transversely-isotropic medium with a vertical axis of symmetry. Here, however, \( A_0, A_1, \) and \( A_2 \) are functions of azimuth \( \phi \) written as

\[ A_0 = \alpha_{00} - \alpha_{20}/2 + 3\alpha_{40}/8 + 3[(2\alpha_{22} - 5\alpha_{42}) \cos 2\phi + (2\beta_{22} - 5\beta_{42}) \sin 2\phi] \]
\[ + 210[\alpha_{44} \cos 4\phi + \beta_{44} \sin 4\phi], \]  

(9)

\[ A_1 = 3\alpha_{20}/2 - 15\alpha_{40}/4 - 6[(\alpha_{22} - 20\alpha_{42}) \cos 2\phi + (\beta_{22} - 20\beta_{42}) \sin 2\phi] \]
\[ - 420[\alpha_{44} \cos 4\phi + \beta_{44} \sin 4\phi], \]  

(10)

\[ A_2 = 35\alpha_{40}/8 - 105[\alpha_{42} \cos 2\phi + \beta_{42} \sin 2\phi] \]
\[ + 210[\alpha_{44} \cos 4\phi + \beta_{44} \sin 4\phi]. \]  

(11)

Substituting equation (8) into equation (1) then gives

\[ r^2(x) = r^2(0) + \frac{x^2}{v_{\text{NMO}}^2} - \frac{A_0^4}{(x^2 + z^2)^2}. \]  

(12)

Here \( r(0) = z/v_V \) is the vertical one-way traveltime, \( v_V \) is the vertical velocity, \( v_{\text{NMO}} \) is the normal-moveout (NMO) velocity, and \( A \) is a measure of the anellipticity of the medium. Parameters \( v_V, v_{\text{NMO}}, \) and \( A \) are given by

\[ v_V = (A_0 + A_1 + A_2)^{-1/2}, \]  

(13)

\[ v_{\text{NMO}} = (A_0 - A_2)^{-1/2}, \]  

(14)

\[ A = -A_2. \]  

(15)

For a transversely-isotropic medium with a vertical axis of symmetry, equation (12) is equivalent to the result of Byun et al. (1989). These authors show that, even for highly anisotropic media, this gives an excellent fit over a wide range of ray angles. Tsvankin and Thomsen (1994) show that if the parameters are calculated from the elastic constants of the medium by assuming the anisotropy to be weak, this equation gives a poor approximation for strong anisotropy. In this paper, the parameters in equation (12) are obtained from a fit to the data, and are not assumed to be related to the elastic constants of the medium by equations derived using the assumption of weak anisotropy.

An estimate of the horizontal velocity, \( v_H \), follows from equation (8)

\[ v_H = A_0^{-1/2}. \]  

(16)

In terms of \( v_{\text{NMO}} \) and \( A \), it follows that

\[ v_H = \frac{v_{\text{NMO}}}{\sqrt{1 - Av_{\text{NMO}}^2}}. \]  

(17)

Note that in equations (13) and (14)

\[ A_0 + A_1 + A_2 = \alpha_{00} + \alpha_{20} + \alpha_{40}, \]  

(18)

which is independent of \( \phi \), and

\[ A_0 - A_2 = \alpha_{00} - \alpha_{20}/2 - 4\alpha_{40} + 6[(\alpha_{22} + 15\alpha_{42}) \cos 2\phi + (\beta_{22} + 15\beta_{42}) \sin 2\phi], \]  

(19)

which depends only on \( \cos 2\phi \) and \( \sin 2\phi \). Thus \( v_{\text{NMO}} \) takes the form

\[ v_{\text{NMO}} = (a_1 + a_2 \cos 2\phi + a_3 \sin 2\phi)^{-1/2}. \]  

(20)

A takes the form [see equation (11)]

\[ A = a_1 + a_5 \cos 2\phi + a_6 \sin 2\phi + a_7 \cos 4\phi + a_8 \sin 4\phi. \]  

(21)

Assuming the depth from source to receiver is known, the coefficients \( \alpha_{00} \) and \( \beta_{00} \) occurring in equation (7) can be obtained by fitting equation (12) to traveltime data acquired at a sufficient number of azimths. The ray velocity then follows from equation (2). This will be illustrated below using ray-traced traveltimes for a fractured shale and measured traveltimes for an azimuthally anisotropic physical model. A least-squares method is employed for the inversion (Lines and Treitel, 1984). Seismic data are usually contaminated by noise. Depending on the characteristics of the noise, other optimization criteria, such as iteratively reweighted least squares (Scales et al., 1988), which are more robust than least squares may be used to estimate the parameters.

For a monoclinic medium, it is seen from equation (7) that a minimum of five azimuths is required to determine the parameters \( \alpha_{00} \) and \( \beta_{00} \) occurring in this equation. If the objective is only to determine the azimuthal variation in \( v_{\text{NMO}} \) then three azimuths are sufficient for noise free data. However, an accurate determination of the azimuthal variation in \( v_{\text{NMO}} \) requires reasonably large source-receiver offsets and therefore requires the nonhyperbolic moveout to be taken into account. Furthermore, a comparison of the variation of \( v_{\text{NMO}} \) with that of \( v_{\text{NMO}} \) gives important information on the properties of the fractures, as will be shown below. In the examples which follow, traveltimes for five lines equally spaced in azimuth will be used. None of the lines are parallel to the symmetry directions of the medium, the objective being to determine these directions from the data.

### FRACTURED SHALE

To illustrate the use of the theory introduced above, consider the case of a horizontal shale layer containing a set of vertical fractures, with normals at 30° to Ox, as shown in Figure 2.

In the absence of fractures the shale is assumed to be transversely isotropic, with symmetry axis along Ox and elastic stiffnesses: \( c_{11} = 19.19, c_{13} = 15.65, c_{11} = 7.06, c_{13} = 4.11, c_{66} = 5.70 \) GPa and density \( \rho = 2.5 \) g/cm³. These values were obtained from the elastic stiffnesses of Greenhorn shale following the procedure described in Appendix A. The fracture set is described by parameters \( Z_{\parallel} \) and \( Z_{\perp} \) (Schoenberg and Sayers, 1995). For gas-filled open fractures \( Z_{\parallel}/Z_{\perp} \approx 1 \), but a lower ratio of \( Z_{\parallel}/Z_{\perp} \) may result from the presence of cement or clay within the fractures, or from the presence of a fluid with nonzero bulk modulus. Two models were considered with elastic constants calculated as described in Appendix B.
Model 1

For this example it was assumed that $Z_T = Z_N$ and $\delta_N = Z_N c_{44}^{04}/(1 + Z_N c_{44}^{04}) = 0.2$. This results in the following elastic stiffness matrix

$$
\begin{bmatrix}
16.11 & 6.28 & 5.86 & 0 & 0 & -0.67 \\
6.28 & 17.71 & 6.28 & 0 & 0 & -0.72 \\
5.86 & 6.28 & 15.13 & 0 & 0 & -0.36 \\
0 & 0 & 0 & 4.06 & -0.09 & 0 \\
0 & 0 & 0 & -0.09 & 3.95 & 0 \\
-0.67 & -0.72 & -0.36 & 0 & 0 & 5.34
\end{bmatrix}
$$

with components in GPa. Note that the difference in $c_{33}$ between the fractured and unfractured shale for this model is caused by the nonzero value of $\delta_N$ [see equation (B-6)].

For gas-saturated cracked porous sandstones the assumption that $Z_N = Z_T$ may not be unreasonable. For dry penny-shaped cracks $Z_N/Z_T = 1 - v_b/2$ (Sayers and Kachanov, 1991) and from measurements on Berea sandstone, for example, $v_b = 0.11$ (Lo et al., 1986). This yields $(Z_T - Z_N)/(Z_T + Z_N) = 0.028$ as a measure of how much $Z_N$ differs from $Z_T$.

Model 2

For this example, it was assumed that $Z_N = 0$ and $\delta_N = Z_T c_{44}^{04}/(1 + Z_T c_{44}^{04}) = 0.2$. This gives the following elastic stiffness matrix

$$
\begin{bmatrix}
18.09 & 8.89 & 7.06 & 0 & 0 & 0.63 \\
8.89 & 18.09 & 7.06 & 0 & 0 & -0.63 \\
7.06 & 7.06 & 15.65 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.90 & -0.36 & 0 \\
0 & 0 & 0 & -0.36 & 3.49 & 0 \\
0.63 & -0.63 & 0 & 0 & 0 & 5.34
\end{bmatrix}
$$

with components in GPa.

A lower ratio of $Z_N/Z_T$ could result from the presence of cement or clay within the fractures, or in the presence of a fluid with nonzero bulk modulus.

Results

Figure 3 shows traveltime squared versus offset squared computed for the two models for a receiver at a depth of 1 km and a maximum source/receiver offset equal to twice the receiver depth. Five lines equally spaced in azimuth were used. None of these lines are parallel to the symmetry directions of the medium, the objective being to determine these directions from the data. The traveltime is nonhyperbolic as a result of the polar anisotropy of the shale.

Figure 4 shows the variation in the small-offset NMO velocity, $v_{NMO}$, and horizontal velocity $v_H$ plotted as a function of azimuth $\phi$ obtained from equations (14) and (16) by fitting

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**Fig. 2.** Homogeneous shale containing a single set of fractures with normals at 30° to $Ox_1$.  

**Fig. 3.** Traveltime squared versus offset squared for a homogeneous shale containing a single set of fractures with normals at 30° to $Ox_1$ for (a) model 1 and (b) model 2. The thickness of the layer is 1 km.
To determine the resolution of the method, the root-mean-square error in the traveltime (Tsvankin and Thomsen, 1995)

$$\Delta t_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \Delta t_i^2}$$

was computed for various \( (V_{\text{NMO}}, V_H) \) pairs for the \( \phi = 0^\circ \) acquisition line. Here \( N \) is the number of source positions, and \( \Delta t \) is the difference between the actual traveltime and that calculated using equation (12). The results are shown in Figure 6 where it is seen that the resolution in \( V_H \) is comparable to that in \( V_{\text{NMO}} \) for the range of offsets used. The root-mean-square traveltime error corresponding to the minimum in this figure is 0.12 ms for model 1 and 0.06 ms for model 2. Equation (12) therefore gives an excellent fit to the data.

The accuracy to which \( V_H \) can be determined decreases as the maximum offset decreases. Figure 7, for example, plots the rms traveltime residuals when the maximum source/receiver offset is equal to the receiver depth, and \( V_H \) is seen to be much less well

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**Fig. 4.** Variation in the small-offset NMO velocity, \( V_{\text{NMO}} \), and horizontal velocity, \( V_H \), for (a) model 1 and (b) model 2 plotted as a function of azimuth \( \phi \) obtained from equations (15) and (16) by fitting equation (12) to the data plotted in Figure 3. The dashed curves show the exact values computed from the elastic stiffnesses of the medium.

**Fig. 5.** Stereographic projection of the ray velocity obtained by fitting the traveltime data in Figure 3 to equation (12) for (a) model 1 and (b) model 2.
resolved in comparison with Figure 6. Although a determination of the orientation of the fractures using $v_{NMO}$ is preferable, a comparison of the variation of $v_H$ with that of $v_{NMO}$ gives important information on the value of $Z_N/Z_T$. Furthermore, the accurate determination of the azimuthal variation in $v_{NMO}$ requires reasonably large source-receiver offsets and therefore requires the nonhyperbolic moveout to be taken into account.

**Acquisition of experimental data**

A physical model (scale-model) simulation of a reverse vertical seismic profile (RVSP), i.e., with a source in the borehole and receivers at the surface, in an anisotropic medium was performed at the Allied Geophysical Laboratory, University of

**FIG. 6.** $P$-wave root-mean-square traveltime residuals (in milliseconds) calculated with respect to the exact values for (a) model 1 and (b) model 2 for various $(v_{NMO}, v_H)$ pairs. The maximum source/receiver offset is 2 km and the receiver depth is 1 km. The zero-offset traveltime is 0.4065 s for (a) and 0.3997 s for (b).

**FIG. 7.** $P$-wave root-mean-square traveltime residuals (in milliseconds) calculated with respect to the exact values for (a) model 1 and (b) model 2 for various $(v_{NMO}, v_H)$ pairs. The maximum source/receiver offset is 1 km and the receiver depth is 1 km. The zero-offset traveltime is 0.4065 s for (a) and 0.3997 s for (b).
was from 0° to 67°. The spacing between each consecutive receiver location was 10 m (unscaled 1 mm), which allowed for adequate sampling of the received wavefield. The receiver-line detector locations (geophone locations in the field) were occupied one at a time until all the desired locations had been occupied. In a serial fashion, this allowed the simulation of the actual field case where all of the receivers are recorded simultaneously.

We used a scale factor of 1:10000 for length, and 1:10000 for time. The advantage of having the length and time scale factors equal was that the velocity scale factor was 1:1. That is, analysis of the traveltime data in either scaled or unscaled units yielded the actual velocity of the medium.

The near horizontal offset for each of the lines was 0 m (unscaled 0 mm), while the far horizontal offset for each of the lines was 1490 m (unscaled 149 mm). Thus, the ray angle range of the experiment (taking vertical as 0°, and horizontal as 90°) was from 0° to 67°. The spacing between each consecutive receiver location was 10 m (unscaled 1 mm), which allowed for adequate sampling of the received wavefield.

The actual medium used was a piece of Phenolite, an industrial laminate commonly used in the manufacture of electrical power grid transformers. Phenolite has been used previously in a number of physical model experiments involving wave propagation in anisotropic media. Uren et al. (1990a, b) and Roldan et al. (1994) studied dip traveltime and zero-offset migration using Phenolite models. Okoye et al. (1995) has examined premigration horizontal seismic resolution in anisotropic media using different pieces of Phenolite with differing elastic constants.

**Fig. 8.** Schematic overview of the experiment.

### Analysis of laboratory data

The experiments were performed with custom-made Harisonsics vertical component transducers. These transducers were designed to have a small active element width to be more representative of seismic measurements. They do, however, have very little backing behind the active element, and so the wavelet had a reverberatory character. This reverberatory character was not a problem in this experiment because only the time of the first arrival of the P-wave was used.

The custom-made transducers had a nominal width of 3.5 mm (35 m scaled). The rounded housing enclosing the active elements slightly reduced the effective widths of the transducers. The data from Line 3 are shown in Figure 9. The P-wave direct arrival is the approximately hyperbolic event with a zero-offset arrival time of slightly more than 0.2 s (20 µs unscaled) and a far-offset time of slightly more than 0.4 s (40 µs unscaled).

Plots of traveltime-squared against offset-squared ($t^2$ versus $x^2$) for the picked times of the direct arriving P-wave for the five lines are shown in Figures 10 and 11. Also shown is a fit of equation (12) to the picked traveltimes. The picked traveltimes are seen to fluctuate randomly about the theoretical curves at large offsets as a result of the lower signal to noise ratio. This is caused by the relatively small amount of energy that can be input into the Phenolite by the custom-made transducers, given the small size of their active elements.

The $t^2$ versus $x^2$ curves shown Figures 10 and 11 are concave downward, in agreement with the expected behavior of layered sedimentary rocks (Sayers, 1995). The azimuthal variation in traveltime is clearly visible. Figure 12 shows a stereographic projection of the ray velocities derived from the analysis. The anisotropic velocity field of the physical model is seen to possess almost perfect orthorhombic symmetry. This result is in accord with the results of earlier physical model studies using Phenolite (Brown et al., 1991, 1993).

An important distinction between these experiments and those of Brown et al., is that Brown and co-workers made explicit use of the knowledge of the symmetry plane locations when collecting data. The present experiments were designed specifically to exclude use of such a priori knowledge (which can be obtained in the case of Phenolite by examining the faces of the block).

The fact that this experiment produced a result consistent with earlier workers can thus be seen as a validation of the analysis technique when applied to circumstances in which the symmetry planes are not known beforehand. In the field, of course, the orientation of any symmetry planes is usually not known. Hence, this technique is applicable to $P$-wave data collected at multiple azimuths using multiple-offset VSP techniques.

The same experiments were also performed with an off-the-shelf pair of vertical component transducers from Harisonsics (model number CMO108-S). These transducers have a much larger active element width of 12.7 mm (127 m scaled). The transducers also have a very effective backing, resulting in a wavelet that has few reverberations. The data collected for line 3 using these transducers are shown in Figure 13. The data are less noisy at large offsets because of the large amount of energy that can be injected with large active element widths.

The picked arrival times from the data shown in Figure 13 are compared in Figure 14 with those obtained for the same line using the smaller transducers. The picked times at large
offsets are seen to have little deviation from a smooth curve, a
direct indication that the data have a high signal-to-noise level
even at large offsets. Unfortunately, the improved signal-to-
noise comes at too high a price. In Figure 14, the curve is seen
to be concave upward, opposite to the behavior found with the
smaller transducers.

The reason that the traveltimes derived from the large trans-
ducer data are distorted is explained easily by looking at array
effects. The response of the large transducers violates the im-

plicit assumption made in most traveltime analyses, namely,
that the source and receiver can be assumed to be point-like
(i.e., having dimensions too small to affect the experiment). The
large transducers had a width equal to about 8% of the
far-offset travel path of 162 mm (1620 m scaled), easily enough
to skew the arrival times.

The array effect of the large transducers can be removed
from the data by reducing the horizontal offset of the curve ob-
tained by fitting the data for the smaller transducers by 10 mm
(100 m scaled), as is illustrated in Figure 14. This is similar to
the shift that Brown et al. (1993) found necessary to recon-
cile their data with the observed traveltimes. These authors
concluded that the effective path length was shorter than the
nominal distance between transducer centers and was in fact
close but not exactly equal to the distance between the nearest
edges of transducers, which constituted a significant difference
for the transducers used.

These two experiments demonstrate that even a straight-
forward traveltime experiment can be adversely affected by
neglecting transducer effects. In this case, the transducer pair
producing what appeared to be lower signal-to-noise data were,
in fact, the preferred transducers for the experiment.

CONFIRMATION OF SYMMETRY AXES
USING SHEAR WAVES

The analysis of the physical model P-wave experimental
traveltine data described above suggests that one of the sym-
metry axes of the block is at a 30° clockwise rotation (seen
from above) from the azimuth of line 1 (see Figure 8). An ad-
ditional symmetry axis is predicted at a 120° clockwise rotation

![Fig. 10. Comparison of the traveltime squared versus offset
squared for data acquired with small transducers with a fit of
equation (12) for lines 1 and 4.](image)

![Fig. 9. Seismic section for line 3 acquired with the small transducers.](image)
from line 1. These symmetry axes would also be encountered at rotations of 210° and 300° clockwise from line 1.

To check these predictions, a rotation experiment using cross-polarized shear-wave transducers was performed. A shear-wave source transducer was coupled to the top surface of the test block with polarization direction parallel to line 1. A shear-wave receiver, directly below the source transducer, was coupled to the lower surface of the test block with the polarization direction at 90° to the source transducer. The two transducers were then rotated together clockwise at 1.8° intervals, so that 200 such intervals returned the transducers to their original orientations. At each intermediate angular position, the source transducer was triggered and the received signal recorded.

For an isotropic medium, and perfect transducers, a zero amplitude signal would result at all azimuthal rotation angles. For an anisotropic medium, and actual transducers, a near-zero amplitude would result when the source transducer is nearly aligned with one of the symmetry axes. At source transducer orientation angles between the symmetry axes, significant amplitudes result, making differential comparison with the results obtained at symmetry axis orientations relatively easy.

In our case, we predicted extinctions at 30°, 120°, 210°, and 300°. Given our initial starting orientation, and an increment of 1.8° per trace, extinctions at trace numbers 18, 68, 118, and 168 are expected using the principal directions obtained from analyzing the P-wave data. This is seen to be the case in Figure 15.

**Fig. 11.** Comparison of the traveltime squared versus offset squared for data acquired with small transducers with a fit of equation (12) for lines 2, 3, and 5.

**Fig. 12.** Stereographic projection of the ray velocity obtained by fitting the traveltime data acquired with the small transducers to equation (12).

**Fig. 13.** Seismic section for line 3 acquired with the large transducers.

**Fig. 14.** Comparison of the traveltime squared versus offset squared for data acquired with small transducers (upper data points) with that for data acquired with the large transducers (lower data points). The upper curve is a fit of equation (12) to the data acquired with small transducers while the lower curve is the same fit but with the horizontal offset x reduced by 1 cm. This modified curve is seen as being in agreement with the data acquired with the large transducers.
A nonhyperbolic traveltime equation, based on an expansion of the inverse-squared ray velocity in spherical harmonics, which can be used in the presence of azimuthally anisotropic layered media, has been presented. This method was applied to the case of horizontal layers having monoclinic symmetry with symmetry plane parallel to the layers. A sedimentary rock containing several sets of fractures with normals lying in the bedding plane is an example of such a medium if, in the absence of fractures, the rock is transversely isotropic with symmetry axis perpendicular to the bedding plane. Using ray-traced traveltimes for a fractured shale, it was shown how the strike of the fractures can be recovered given traveltimes data at sufficiently many azimuths. The theory was applied to a physical model simulation of a reverse vertical seismic profile in an anisotropic medium. Five lines at different azimuths were acquired over a homogeneous anisotropic medium. Inversion of the measured traveltimes showed the medium to be orthorhombic in agreement with previous work. The orientation of the principal axes obtained were in agreement with those given by an independent shear-wave experiment using cross-polarized transducers.

In contrast to previous work, no knowledge of the orientation of the symmetry planes is required. The method is therefore applicable to P-wave data collected at multiple azimuths using multiple offset VSP techniques. It should be noted, however, that the effects of heterogeneity have been neglected. If the lateral heterogeneity can be neglected, as is the case for the field data case of Miller et al. (1994), the interval properties between different receiver depths may be obtained from equation (12) by expanding the traveltime in powers of horizontal slowness and by using the fact that horizontal slowness is conserved for a depth dependent, but laterally homogeneous, medium (Dellinger et al., 1993). This approach is described in Appendix C, and requires the azimuthal anisotropy to be sufficiently small so that the deviation of the rays from the vertical plane containing source and receiver may be neglected.

If the lateral heterogeneity is small, perturbation theory may be used to extend the present results. Only the traveltimes vary to first-order in the lateral heterogeneity, the variation in the raypaths being of second-order. Larger lateral variations may restrict the range of useful offsets and therefore reduce the ability to estimate $v_H$ from the data. Note, however, that ray bending in the overburden may allow good angular coverage in the zone of interest. For example, Miller et al. (1994) found that for a receiver at a depth of 1 km, turning rays (i.e., rays traveling horizontally at the receiver) occurred at source-receiver offsets of about 2 km either side of the well. The data display a high degree of symmetry about zero offset, suggesting that the medium is laterally homogeneous. It should be noted that the form of equation (20) holds even in the presence of lateral heterogeneity provided that the offsets are measured with respect to the minimum traveltime location at the surface. This follows by expanding the traveltime as a Taylor series in offset $x$ and using the fact that the first term is of second-order in $x$ in the vicinity of the minimum traveltime location (Hubral and Krey, 1980; Grechka and Tsvankin, 1996).

![Fig. 15. Confirmation of symmetry axes of experimental block using shear waves.](image-url)
We thank Luc Ikelle for facilitating the transfer of data from Houston to Cambridge.

REFERENCES


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APPENDIX A

ELASTIC STIFFNESSES FOR SHALES

In the absence of fractures, the shale in the model is assumed to be transversely isotropic with a vertical axis of symmetry. If the symmetry axis is chosen as the $z_3$-direction, the nonzero elastic stiffnesses $C_{ij}$ are $C_{11} = C_{22}, C_{33}, C_{44} = C_{55}, C_{66}, C_{13} = C_{23}$, and $C_{12} = (C_{11} - 2C_{66})$. It is convenient to write the $C_{ij}$ in the form

$$C_{ij} = C_{ij}^0 + \Delta_{ij}, \quad (A-1)$$

where $C_{ij}^0$ are the elastic stiffnesses of an isotropic comparison medium, and $\Delta_{ij}$ is the difference between $C_{ij}^0$ and $C_{ij}$. The isotropic comparison model was chosen as the isotropic medium most similar to the anisotropic medium in the sense of Fedorov (1968) and has second-order Lame elastic constants given by

$$15\lambda = C_{11} + C_{22} + 4C_{33} + 4C_{12} + 4C_{13} - 2C_{44} - 2C_{55} - 2C_{66}, \quad (A-2)$$

$$15\mu = C_{11} + C_{22} + C_{33} - C_{12} - C_{23} - C_{13} + 3C_{44} + 3C_{55} + 3C_{66}. \quad (A-3)$$

The anisotropic velocity of the shale is assumed to be that given by choosing the $\Delta_{ij}$ to be half the values calculated for Greenhorn shale using the isotropic comparison medium of Fedorov.
The effective elastic compliance tensor $s_{ijkl}$ of a rock containing fractures relates the average strain $\varepsilon_{ij}$ over a representative volume $V$ to the average stress components $\sigma_{kl}$:

$$\varepsilon_{ij} = s_{ijkl}\sigma_{kl}. \quad (B-1)$$

For fractures, $\varepsilon_{ij}$ may be written in the form,

$$\varepsilon_{ij} = s_{ijklb}\sigma_{kl} + \frac{1}{2V} \sum_q \int_{S_q} ([u_i]n_j + [u_j]n_i) dS, \quad (B-2)$$

where $s_{ijklb}$ is the compliance tensor of the unfractured background rock, which may be of arbitrary anisotropy, $S_q$ is the surface of the $q$th fracture lying within $V$, $n_i$ are the components of the local unit normal to the fracture surface, which may generally be curved, and brackets $[,]$ denote jump discontinuities in the displacement; see, for example, Sayers and Kachanov (1991). Equation (B-2) is applicable to finite, non-planar fractures in the long wavelength limit, i.e., the applied stress caused by the seismic wave is assumed to be constant over the representative volume $V$.

$$\begin{bmatrix}
  c_{11b}(1 - \delta_N) & c_{12b}(1 - \delta_N) & c_{13b}(1 - \delta_N) & 0 & 0 & 0 \\
  c_{12b}(1 - \delta_N) & c_{11b}(1 - \delta_N - \frac{c_{12b}^2}{c_{11b}}) & c_{13b}(1 - \delta_N - \frac{c_{12b}^2}{c_{11b}}) & 0 & 0 & 0 \\
  c_{13b}(1 - \delta_N) & c_{13b}(1 - \delta_N - \frac{c_{12b}^2}{c_{11b}}) & c_{33b}(1 - \delta_N - \frac{c_{12b}^2}{c_{11b}}) & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}, \quad (B-6)$$

For a single fracture set having unit normal $n$ with components $n_i$, a linearity assumption is conveniently introduced through a “fracture system compliance tensor” $Z$ with components $Z_{ij}$ such that,

$$\frac{1}{V} \sum_q \int_{S_q} [u_i] dS = Z_{ijkl} n_k. \quad (B-3)$$

where $Z_{ij}$ is symmetric and non-negative definite (Schoenberg and Sayers, 1995).

It will be assumed that the normal compliance of the fractures is given by $Z_N$ and the tangential compliance by $Z_T$ so that

$$Z_{ij} = Z_N n_i n_j + Z_T (\delta_{ij} - n_i n_j). \quad (B-4)$$

It then follows that the excess compliance tensor of the fracture set is given by Schoenberg and Sayers (1995)

$$s_{ijkl} = \frac{Z_T}{4} \left( \delta_{ik} n_i n_j + \delta_{jk} n_i n_i + \delta_{il} n_k n_j + \delta_{jl} n_k n_i \right) + (Z_N - Z_T) n_i n_j n_k. \quad (B-5)$$

The elastic stiffness tensor of the fractured medium is obtained by inverting the compliance tensor obtained by adding the excess compliance tensor caused by the fractures to the compliance tensor of the shale. For a TI background medium with a single set of aligned vertical fractures perpendicular to the 1-direction, the elastic stiffness matrix that results is

$$\begin{bmatrix}
  Z_{Nc_{11b}} & 0 & 0 & 0 \\
  0 & Z_{Tc_{44b}} & 0 & 0 \\
  0 & 0 & Z_{Tc_{66b}} & 1 - \delta_H \\
  0 & 0 & 0 & 0 
\end{bmatrix}, \quad (B-7)$$

where

$$0 \leq \delta_N = \frac{Z_N c_{11b}}{1 + Z_N c_{11b}} < 1, \quad 0 \leq \delta_T = \frac{Z_T c_{44b}}{1 + Z_T c_{44b}} < 1, \quad 0 \leq \delta_H = \frac{Z_T c_{66b}}{1 + Z_T c_{66b}} < 1,$$

in agreement with the results of Hood and Schoenberg (1989).
APPENDIX C
INTERVAL VELOCITY ANALYSIS

It will be assumed that the azimuthal anisotropy is sufficiently small that the deviation of the rays from the vertical plane containing source and receiver can be neglected. If this is the case, the interval properties between different receiver depths, for a walkaway VSP acquired with receivers at several depths in the borehole, may be obtained from equation (12) by expanding the traveltime in powers of horizontal slowness and by using the fact that horizontal slowness \( p \) is conserved for a depth dependent, but laterally homogeneous, medium (Dellinger et al., 1993). This gives

\[
t(p) = t(0)\left[1 + \frac{p^2 v_{NMO}^2}{2} + \frac{3p^4 v_{NMO}^4}{8}\right]
\times \left(1 + \frac{4A r^2(0)v_{NMO}^4}{z^2} \right) + \ldots \right].
\]  

(C-1)

This allows the stack properties to be calculated from the properties of the different layers to obtain

\[
t_{\text{stack}}(0) = \sum t_i(0), \quad \text{(C-2)}
\]

\[
t_{\text{stack}}(0)v_{\text{NMO}}^2_{\text{stack}} = \sum t_i(0)v_{\text{NMO}}^2_i, \quad \text{(C-3)}
\]

\[
t_{\text{stack}}(0)v_{\text{NMO}}^4_{\text{stack}} \left(1 + \frac{4A r^2(0)v_{\text{NMO}}^4_{\text{stack}}}{z_{\text{stack}}^2} \right)
= \sum t_i(0)v_{\text{NMO}}^4_i \left(1 + \frac{4A r^2(0)v_{\text{NMO}}^4_i}{z_i^2} \right). \quad \text{(C-4)}
\]

Equation (C-2) expresses the additivity of vertical traveltimes, while equation (C-3) is the rms-velocity equation of Dix (1955). Equation (C-4) allows the anellipticity parameter of each layer to be determined.