Effects of borehole stress concentration on elastic wave velocities in sandstones

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Summary

The stress redistribution that occurs in the vicinity of a borehole may lead to damage or failure of the rock. Recent developments in sonic logging allow the variation in elastic wave velocities around a borehole to be investigated. Since elastic wave velocities in sandstones are sensitive to changes in stress due to the presence of stress-sensitive grain boundaries within the rock, this allows the changes in stress to be monitored. For small changes in stress, perturbation theory shows that a radially polarized, vertically propagating shear wave is more sensitive to radial stress than to hoop stress. Close to the wellbore large changes in stress occur. Non-linearity in the variation in velocity with stress increases the sensitivity of wave velocities to the wellbore pressure, and can cause the velocity of both compressional and shear waves to be significantly lower than that predicted by perturbation theory.

Introduction

The drilling of wells leads to significant changes in the stress field in the vicinity of the borehole, and these changes in stress may lead to damage or failure of the rock. Recent developments in sonic logging have made it possible to map the 3D variation in elastic wave velocities around the borehole (Pistre et al., 2005). In this paper, the effect of the stress redistribution on the velocity of vertically propagating elastic waves is investigated.

Figure 1 shows the variation in effective vertical, radial and hoop stress at radial distance $r$ from a borehole of radius $a$ in directions parallel ($\phi = 0^\circ$) and perpendicular ($\phi = 90^\circ$) to the maximum far-field horizontal stress for the example given in the text.

Figure 1. Variation in vertical, radial and hoop stress at radial distance $r$ from a borehole of radius $a$ in directions parallel ($\phi = 0^\circ$) and perpendicular ($\phi = 90^\circ$) to the maximum far-field horizontal stress for the example given in the text.

Stress-dependence of elastic wave velocities

It is assumed that the elastic wave velocities are a function of the effective stress tensor, $\sigma_{ij}$, which is assumed to be given in terms of the total stress tensor, $S_{ij}$, and the pore pressure, $p$, by

$$\sigma_{ij} = S_{ij} - p\delta_{ij}; \quad (1)$$

where $\delta_{ij}$ is the Kronecker delta, and $\delta_{ij} = 1$ if $i=j$ and 0 otherwise. Elastic wave velocities in sandstones vary with changes in effective stress due to the presence of stress-sensitive grain boundaries within the rock. Sayers and Kachanov (1995) show that the elastic compliance tensor, $S_{ijkl}$, of a sandstone may be written in the form

$$S_{ijkl} = S_{ijkl}^0 + \Delta S_{ijkl}; \quad (2)$$

where $S_{ijkl}^0$ is the compliance that the rock would have if the rock grains formed a continuous framework, and $\Delta S_{ijkl}$ is the excess compliance due to the presence of the grain boundaries in the rock. The term $\Delta S_{ijkl}$ can be written as

$$\Delta S_{ijkl} = \frac{1}{4} \left( \delta_{ik}\alpha_{jl} + \delta_{il}\alpha_{jk} + \delta_{jk}\alpha_{il} + \delta_{jl}\alpha_{ik} \right) + \beta_{ijkl} \quad (3)$$

(Sayers and Kachanov, 1995), where $\alpha_{ij}$ is a second-rank tensor and $\beta_{ijkl}$ is a fourth-rank tensor defined by
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\[
\alpha_{ij} = \frac{1}{V} \sum_r B^{(r)}_T n_i^{(r)} n_j^{(r)} A^{(r)},
\]
(4)

\[
\beta_{ijkl} = \frac{1}{V} \sum_r \left( B^{(r)}_N - B^{(r)}_T \right) n_i^{(r)} n_j^{(r)} n_k^{(r)} n_l^{(r)} A^{(r)}.
\]
(5)

The terms \( B^{(r)}_N \) and \( B^{(r)}_T \) are the normal and shear compliance of the \( r \)th grain boundary; \( n_i^{(r)} \) is the area of the normal to the grain boundary; \( A^{(r)} \) is the component of the normal to the grain boundary; \( V \) is the volume of the grain boundary; and the summation in equations (4) and (5) is over all grain boundaries lying within volume \( V \). If the normal and shear compliance of the discontinuities are equal, it follows from equation (5) that the fourth-rank tensor \( \beta_{ijkl} \) vanishes, and the elastic stiffness tensor is a function only of \( \alpha_{ij} \). This is a reasonable approximation for the grain contacts in sandstones (Sayers, 2002) and will be assumed in the remainder of this paper.

Following Mavko et al. (1995) and Schoenberg (2002), it is assumed that the normal and shear compliance of a grain boundary are functions only of the component of the effective stress acting normal to the plane of the boundary given by \( \sigma_n = n_i \sigma_{ij} n_j \), where a sum over repeated indices is implied. The components \( n_i \) of the normal \( n \) to a grain boundary can be written in terms of the polar angle \( \theta \) and the azimuthal angle \( \phi \) relative to a set of axes \((x_1, x_2, x_3)\) fixed in the rock:

\[
n_1 = \cos \phi \sin \theta, \; n_2 = \sin \phi \sin \theta \quad \text{and} \quad n_3 = \cos \theta.
\]
(6)

The normal component \( \sigma_n \) of the stress acting on a grain boundary is then given by

\[
\sigma_n = n_i \sigma_{ij} n_j = \sigma_1 \cos^2 \phi \sin^2 \theta + \sigma_2 \sin^2 \phi \sin^2 \theta + \sigma_3 \cos^2 \theta.
\]
(7)

Assuming a continuous orientation distribution of discontinuities, \( \alpha_{ij} \) can be written in the form

\[
\alpha_{ij} = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} Z(\theta, \phi) n_i n_j \sin \theta \sin \phi d\phi d\theta.
\]
(8)

where \( Z(\theta, \phi) \sin \theta d\phi d\theta \) represents the compliance of all discontinuities with normals in the angular range between \( \theta \) and \( \phi \) and \( \theta \) and \( \phi \) in a reference frame \( X_1X_2X_3 \) with axis \( X_3 \) aligned with the normal to the discontinuity.

Han (1986) measured compressional and shear wave velocities on 24 room-dry Gulf of Mexico sandstones; the results for the sample with the greatest stress sensitivity is shown in Figure 2. The rate of increase in velocity with increasing stress decreases with increasing stress. This is consistent with the expected decrease in the compliance of the grain boundaries as the stress is increased due to increasing contact between opposing faces of the grain boundary. Following Schoenberg (2002), it is assumed that the compliance of the grain boundaries decreases exponentially with increasing stress applied normal to the grain boundaries as follows:

\[
Z = Z_0 e^{-\sigma_c/\sigma_0},
\]
(9)

where \( \sigma_0 \) is a characteristic stress that determines the rate of decrease.

Figure 2. Measured ultrasonic \( P \)- and \( S \)-wave velocities in a room-dry Gulf of Mexico sandstone as a function of increasing hydrostatic stress (Han, 1986). The curves show a fit of equations (2-4) using the exponential variation of equation (9).

The elastic stiffness tensor can be found by inverting the compliance tensor given by equations (2-4). This allows the elastic wave velocities to be calculated. The curves in Figure 2 show a fit of the theory to the data shown using equation (9) and neglecting the contribution of \( \beta_{ijkl} \). The exponential form of equation (9) gives a good fit to the data and yields the following parameters: \( \sigma_0 = 10.3 \) MPa and \( \mu Z_0 = 0.438 \), where \( \mu \) is the shear modulus at a confining stress of 50 MPa. The average values of \( \sigma_e \) and \( \mu Z_0 \) for the 24 room-dry Gulf of Mexico sandstones studied by Han (1986) are \( \sigma_e = 10.13 \) MPa and \( \mu Z_0 = 0.2583 \) respectively.

Example

To illustrate the results, changes in the elastic wave velocities were computed for the values of \( \sigma = 10 \) MPa and \( \mu Z_0 = 0.25 \), which are close to the average values obtained for the 24 room-dry Gulf of Mexico sandstones studied by Han (1986).
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For dry sandstones, Han (1986) found the following variation of P- and S-wave velocity with porosity, φ and clay content C at a confining stress of 40 MPa:

\[
v_p (\text{km/s}) = 5.41 - 6.35\phi - 2.87C \quad (10)
\]

\[
v_s (\text{km/s}) = 3.57 - 4.57\phi - 1.83C \quad (11)
\]

The components \( S_{ij}^{(n)} \) in equation (2) were computed from equations (10) and (11) for the case of a clean gas sand with a porosity of 0.2. This gives a Poisson ratio of 0.15. The values of the vertical, maximum horizontal, and minimum horizontal components of the far-field total stress; the pore pressure; and the pressure exerted on the borehole wall by the fluid in the borehole are assumed to take the same values as those used for Figure 1. For these parameters, Figure 3 shows the radial variation in the predicted velocity of the vertically propagating, radially polarized shear wave at azimuths of \( \phi = 0^\circ \) and \( \phi = 90^\circ \) obtained using equation (9). Also shown are the velocity values derived from a perturbation theory, which is valid for small changes in stress.

The perturbation theory shown in Figure 3 is obtained by writing \( \sigma_n = \sigma_n^{(0)} + \Delta \sigma_n \), where \( \sigma_n^{(0)} \) is the value of \( \sigma_n \) in the initial state of the reservoir, and \( \Delta \sigma_n \) is the change in \( \sigma_n \) due to production. It follows that, for small changes in stress,

\[
Z(\sigma_n) = Z_n^{(0)} + Z_n^{(1)}(\Delta \sigma_n) \quad (12)
\]

where \( Z_n^{(0)} = Z(\sigma_n^{(0)}) \), and \( Z_n^{(1)} \) is the first derivative of \( Z \) with respect to \( \sigma_n \), evaluated at \( \sigma_n^{(0)} \). The non-vanishing components of the change \( \Delta \sigma_{ij} \) in \( \sigma_n \) are \( \Delta \epsilon_{11} \), \( \Delta \epsilon_{22} \), and \( \Delta \epsilon_{33} \), given by

\[
\Delta \epsilon_{11} = \frac{2\pi}{15}(3\Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3)Z_T^{(1)} \quad (13)
\]

\[
\Delta \epsilon_{22} = \frac{2\pi}{15}(\Delta \sigma_1 + 3\Delta \sigma_2 + 3\Delta \sigma_3)Z_T^{(1)} \quad (14)
\]

\[
\Delta \epsilon_{33} = \frac{2\pi}{15}(\Delta \sigma_1 + \Delta \sigma_2 + 3\Delta \sigma_3)Z_T^{(1)} \quad (15)
\]

from which the change in the elastic stiffness tensor can be calculated. It is found that for small changes in stress, the velocity of a vertically propagating compressional wave depends on the change in the radial and hoop stress only through the combination \( \Delta \sigma_n + \Delta \sigma_{n\phi} \), and that the velocity of a vertically propagating, radially polarized, shear wave depends on the change in the vertical, radial and hoop stress only through the combination \( 2(\Delta \sigma_{rr} + \Delta \sigma_{zz}) + \Delta \sigma_{\phi \phi} \). The velocity of a vertically propagating, radially polarized, shear wave is seen to be more sensitive to changes in the vertical and radial stress than to changes in the hoop stress.

Figure 3 – Predicted velocity of vertically propagating, radially polarized shear waves as a function of distance from the borehole wall at azimuths 0° and 90° from the maximum horizontal far-field stress direction using (a) the exponential decrease in compliance given by equation (9), and (b) the perturbation theory of equations (13-15) for the example in the text.

The velocity of radially polarized shear waves propagating along the borehole can be obtained by inversion of borehole flexural wave dispersion data obtained using a dipole sonic tool (Burrigde and Sinha, 1996). The velocities of vertically propagating, radially polarized, shear waves at azimuths \( \phi=0^\circ \) and \( \phi=90^\circ \) (shown in Figure 3) are seen to be strong functions of radius, and display a crossover as the distance from the borehole increases. A crossover in radially polarized shear waves was first predicted by Sinha and Kostek (1996) using third-order elasticity theory. Based on the inversion of borehole flexural wave dispersion curves obtained using a dipole sonic tool, Sinha et al. (2002) reported a crossover in vertically propagating, radially polarized shear wave velocities for sandstones as radius increased from a borehole.

It is seen in Figure 3 that perturbation theory becomes inaccurate close to the wellbore, where large changes in stress may occur. In the near-wellbore region the nonlinearity in the variation of the compliance of the grain boundaries as a function of stress needs to be taken into account, particularly at low wellbore pressure. In contrast to the predictions of perturbation theory, Figure 3 shows that the velocity of radially polarized shear waves for the example decreases near the borehole for both azimuths,
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despite the increase in compressive hoop stress occurring at the azimuth corresponding to the minimum far-field horizontal stress direction.

In the presentation, predicted results for the P-wave velocity will also be shown. It is found that compressional waves may also have velocities that are significantly lower than those predicted by perturbation theory, particularly for low wellbore pressures.

Conclusion

The drilling of wells leads to stress changes in the near-wellbore region; these changes in the stress field lead to changes in elastic wave velocity that may be used to monitor the stress changes that occur. For small changes in stress, perturbation theory predicts that a radially polarized, vertically propagating shear wave is more sensitive to changes in the radial and vertical stress than to changes in the hoop stress. Close to the wellbore large changes in stress may occur, and the nonlinearity in the stress dependence of the compliance of the grain boundaries needs to be taken into account. This nonlinearity occurs because the number of contacts between opposing faces of a grain boundary increases with increasing normal stress, and this increases the sensitivity of wave velocities to wellbore pressure. As a result, the velocity of radially polarized shear waves can decrease rapidly as the borehole wall is approached. Nonlinearity is also important for compressional waves, which can have velocities that are significantly lower than predicted by perturbation theory, particularly for low wellbore pressures.

It follows that it is important to account for non-linear variations in velocity with stress that may occur close to a borehole, where changes in stress may be large. The inclusion of nonlinear effects on compressional and shear wave velocities in the vicinity of the borehole is expected to result in improved techniques for monitoring stress changes around boreholes using sonic logging. The possibility of calibrating the velocity-vs-effective-stress relation using advanced sonic logging techniques may also extend to the calibration of the time-lapse seismic response to changes in reservoir pressure. This may find application in distinguishing between saturation and pore pressure variations in 4D reservoir monitoring.

References


EDITED REFERENCES
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REFERENCES