SUMMARY

Natural fractures in reservoirs, and in the caprock overlying the reservoir, play an important role in determining fluid flow during production, and hence the density and orientation of sets of fractures is of great interest. Chang and Gardner have suggested that the fracture orientation of a subsurface fracture zone may be determined using P-wave interval velocities. In the general case of a rock possessing both an anisotropic fabric and a preferred orientation of fractures, the rock will display both polar and azimuthal anisotropy. A non-hyperbolic moveout equation, based on an expansion of the inverse-squared group velocity in spherical harmonics, is presented which can be used in the presence of azimuthally anisotropic layered media. The theory is applied to the case of horizontal layers having monoclinic symmetry with symmetry plane parallel to the layers. A sedimentary rock containing several sets of fractures with normals lying in the bedding plane is an example of such a medium if, in the absence of fractures, the rock is transversely isotropic with symmetry axis perpendicular to the bedding plane.

INTRODUCTION

Sedimentary rocks frequently possess an anisotropic structure resulting, for example, from fine scale layering, the presence of oriented microcracks or fractures or the preferred orientation of nonspherical grains or anisotropic minerals. Failure to account for anisotropy in seismic processing may lead to errors in velocity analysis, NMO, DMO, time migration, time-to-depth conversion and AVO analysis. Many sedimentary rocks may be described, to a good approximation, as being transversely isotropic (TI) with symmetry axis oriented perpendicular to the bedding plane. Although reflection moveout has been extensively studied for TI media (Hake et al., 1984; Byun et al., 1989; Byun and Corrigan, 1990; Dellinger et al., 1993; Sena, 1991; Tsvankin and Thomsen, 1994), many formations contain fractures with orientations determined by the stress history of the rock rather than by the orientation of the bedding plane. Any fractures open at depth will tend to be oriented normal to the direction of the least compressive in-situ stress.

In the general case of a rock possessing both an anisotropic fabric and a preferred orientation of fractures, the rock will display an azimuthal anisotropy. For such rocks, observations of the seismic anisotropy have the potential of providing the orientation of the least compressive in-situ stress direction. For example, modelling of shear waveforms in three component shear wave vertical seismic profiles in the Paris Basin (Crampin et al., 1986; Bush and Crampin, 1991) is consistent with a distribution of vertical fluid-filled cracks aligned with strikes along N30°W, corresponding to the direction of maximum horizontal in-situ stress in this area. Chang and Gardner (1992) have suggested that the fracture orientation of a subsurface fracture zone may be determined by analysing P-wave interval velocities. Paul (1993) attributed anomalously low stacking velocities observed on the flanks of the Nan Yi Shan anticline to the presence of localized fracturing. The purpose of this paper is to present a non-hyperbolic moveout equation which can be used for velocity analysis in the presence of azimuthally anisotropic layered media.

REFLECTION MOVEOUT FOR AZIMUTHALLY ANISOTROPIC LAYERS

It is convenient to choose a reference set of axes $\mathbf{Oz}_1z_2z_3$ with $\mathbf{Oz}_3$ vertically down (see figure 1). A plan view of the seismic line in figure 1(a) is shown in figure 1(b) and lies at an angle $\phi$ with respect to $\mathbf{Oz}_1$. Since the medium is assumed to be azimuthally anisotropic, the group velocity $V(\theta, \phi)$ is a function of both $\theta$ and $\phi$. Consider the case of a horizontal reflector in an anisotropic medium with a horizontal plane of mirror symmetry. Assuming that the reflection point lies within the vertical plane passing through the source and receiver, it follows that

$$t^2(x) = \frac{x^2}{V^2(\theta, \phi) \cos^2(\theta)},$$

(1)

where $t(x)$ is the one-way travel time at source-receiver half-

Figure 1: (a) Assumed ray geometry for a horizontal reflector in an anisotropic, homogeneous layer having a horizontal plane of mirror symmetry. (b) Plan view of the seismic line shown in (a).
Anisotropic reflection moveout

Figure 2: Homogeneous snare containing a single set of fractures with normals at 30° to $O_x$.

offset $x$ and $z$ is the depth of the reflector. This assumption is usually accurate for sedimentary rocks having a horizontal plane of reflection symmetry. Consider the example of a shale layer containing a single set of fractures with normals at 30° to $O_x$, as shown in figure 2. In the absence of fractures the shale is assumed to be TI with the following elastic stiffnesses: $c_{T1} = 19.19$, $c_{T3} = 15.65$, $c_{S3} = 7.06$, $c_{S5} = 4.11$, $c_{S6} = 5.70$ GPa and density $\rho = 2.5$g/cc. These values were obtained from the elastic stiffnesses of Greenhorn shale (Jones and Wang, 1981) by choosing the anisotropy of the shale to be half that of Greenhorn shale following the procedure described elsewhere (Sayers, 1994a). The fracture set is described by parameters $Z_N$ and $Z_T$ (Schoenberg and Sayers, 1995) with magnitudes given by $Z_T = Z_N$ and $\delta_N = Z_N c_{T1}/(1 + Z_N c_{T1}) = 0.2$. The non-vanishing elastic constants of the fractured shale that result are $c_{T1} = 16.11$, $c_{T2} = 17.71$, $c_{T3} = 15.13$, $c_{T4} = 6.28$, $c_{T6} = 5.86$, $c_{S3} = 6.28$, $c_{S4} = 4.06$, $c_{S6} = 3.95$, $c_{S8} = 5.34$, $c_{S10} = -0.67$, $c_{S26} = -0.72$, $c_{S36} = -0.36$ GPa. Assuming the shale to be 1 km thick, figure 3 shows travel time squared versus offset squared computed for reflections from the bottom of the layer for a maximum source/receiver offset equal to twice the thickness of the layer and five different azimuths measured with respect to $O_x$. Also shown are the values computed using the assumption that the reflection point lies within the vertical plane passing through the source and receiver. Despite the large azimuthal anisotropy, the difference is less than the thickness of the lines drawn. The moveout is seen to be non-hyperbolic as a result of the polar anisotropy of the shale.

EXPANSION IN SPHERICAL HARMONICS

It is convenient to introduce a function $r(\theta, \phi)$ defined by:

$$r(\theta, \phi) = 1/V^2 \theta(\theta, \phi).$$

(2)

$r(\theta, \phi)$ may be expanded as a linear combination of spherical harmonics with expansion coefficients $R_{lm}$ (Sayers, 1988; Sayers, 1994b):

$$r(\xi, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} R_{lm} P_l^m(\xi) \exp(-im\phi),$$

(3)

where $\xi = \cos \theta$. $P_l^m(\xi)$ is the associated Legendre function and $P_l^m(\xi) = (-1)^m P_l^m(\xi)$ with $m = -m$ (Roe 1965). Since the P-wave group velocities are centrosymmetric,

$$r(\xi, \phi) = r(\xi, \phi + \pi),$$

(4)

Expanding both sides of this equation in spherical harmonics and utilizing the relation $P_l^m(-\xi) = (-1)^m P_l^m(\xi)$ gives $R_{lm} = (-1)^m R_{lm}$. This requires $R_{lm}$ to be identically zero when $l$ is odd (Roe 1965).

It is convenient to write $R_{lm}$ in the form:

$$R_{lm} = \alpha_{lm} + i\beta_{lm},$$

(5)

Since the group velocity $V(\theta, \phi)$ is a real quantity, the following relationships hold: $\alpha_{lm} = \alpha_{lm}$, $\beta_{lm} = -\beta_{lm}$ where $m = -m$. Hence $\beta_{lm} = 0$ for $m = 0$. Equation (3) may therefore be written in the form:

$$r(\xi, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} R_{lm} P_l^m(\xi) [\alpha_{lm} \cos m\phi + \beta_{lm} \sin m\phi].$$

(6)

The moveout equation for arbitrary anisotropy is then obtained by substituting equation (6) into equation (1). Equation (6) simplifies in the presence of material symmetry. In the following, attention is restricted to the case of layers having monoclinic symmetry. A material with monoclinic symmetry has a plane of mirror symmetry. A sedimentary rock containing several sets of fractures with normals lying in the bedding plane is an example of a medium with a single plane of mirror symmetry if, in the absence of fractures, the rock is transversely isotropic with symmetry axis perpendicular to the bedding plane. If $O_x$ is normal to the mirror plane, it follows that $r(\xi, \phi) = r(-\xi, \phi)$ and that $R_{lm} = 0$ if
Figure 4: Variation in the small-offset NMO velocity, $v_{NMO}$, and horizontal velocity, $v_H$, plotted as a function of azimuth $\phi$ obtained from equations (15) and (16) by fitting equation (12) to the data plotted in figure 3.

$m$ is odd. Equation (6) then reduces to the following form:

$$
\rho(\xi, \phi) = a_{00} P^2_0(\xi) + a_{20} P^2_2(\xi) + 2(\alpha_{22} \cos 2\phi + \beta_{22} \sin 2\phi) P^2_2(\xi) + \\
2(\alpha_{40} P^4_0(\xi) + 2(\alpha_{42} \cos 2\phi + \beta_{42} \sin 2\phi) P^4_2(\xi) + \\
2(\alpha_{44} \cos 4\phi + \beta_{44} \sin 4\phi) P^4_4(\xi). \quad (7)
$$

Here only terms up to order $l = 4$ are included. Equation (7) does, however, give a good representation even for relatively large anisotropy.

Equation (7) may be written in the form:

$$
\rho(\xi, \phi) = A_0 + A_1 \cos^2 \theta + A_2 \cos^4 \theta, \quad (8)
$$

which is of the same form as that derived by Byun et al. (1989) for a transversely-isotropic medium with a vertical axis of symmetry. Here, however, $A_0$, $A_1$ and $A_2$ are functions of azimuth $\phi$ as follows:

$$
A_0 = a_{00} - a_{20}/2 + 3a_{40}/8 + \\
3[(2\alpha_{22} - 5\alpha_{42}) \cos 2\phi + (2\beta_{22} - 5\beta_{42}) \sin 2\phi] + \\
210[\alpha_{44} \cos 4\phi + \beta_{44} \sin 4\phi], \quad (9)
$$

$$
A_1 = 3a_{20}/2 - 5a_{40}/4 - 6[(\alpha_{22} - 3\alpha_{42}) \cos 2\phi + \\
(\beta_{22} - 3\beta_{42}) \sin 2\phi] - \\
420[\alpha_{44} \cos 4\phi + \beta_{44} \sin 4\phi], \quad (10)
$$

$$
A_2 = 35a_{40}/8 - 105[\alpha_{42} \cos 2\phi + \beta_{42} \sin 2\phi] + \\
210[\alpha_{44} \cos 4\phi + \beta_{44} \sin 4\phi]. \quad (11)
$$

Substituting equation (8) into equation (1) then gives

$$
t^2(x) = t^2(0) + \frac{x^2}{v_{NMO}} - \frac{\alpha x^4}{v_{NMO}^2 (x^2 + z^2)}. \quad (12)
$$

Here $t(0) = z/v_V$ is the vertical one-way travel time and $a$ is the normalized anellipticity parameter defined by

$$
a = 1 - \frac{v_{NMO}}{v_H}. \quad (13)
$$

For transversely isotropic media, $a$ is simply related to the parameter $\eta = \frac{1}{2}(v_H^2/v_{NMO}^2 - 1)$ introduced by Alkhalifah and Tsvankin (1994).

The vertical, $v_V$, horizontal, $v_H$, and normal moveout, $v_{NMO}$, velocities in equation (13) are given by

$$
v_V = (A_0 + A_1 + A_2)^{-1/2}, \quad (14)
$$

$$
v_{NMO} = (A_0 - A_2)^{-1/2}, \quad (15)
$$

$$
v_H = A_0^{-1/2}. \quad (16)
$$

Note that in equations (14) and (15)

$$
A_0 + A_1 + A_2 = a_{00} + a_{20} + a_{40}, \quad (17)
$$

which is independent of $\phi$, and

$$
A_0 - A_2 = a_{00} - a_{20}/2 - 4a_{40}/6[(\alpha_{22} + 15\alpha_{42}) \cos 2\phi + \\
(\beta_{22} + 15\beta_{42}) \sin 2\phi], \quad (18)
$$

which depends only on $\cos 2\phi$ and $\sin 2\phi$.

**INVERSION OF TRAVEL TIMES**

Assuming the depth to the reflector as known, the coefficients $a_{1m}$ and $\beta_{1m}$ occurring in equation (7) can be obtained by fitting equation (12) to traveltime data acquired at a sufficient
Anisotropic reflection moveout

Figure 6: Stereographic projection of the group velocity obtained by fitting the travel time data in figure 3 to equation (12).

number of azimuths. The group velocity then follows from equation (2). Figure 4 shows the variation in the small-offset NMO velocity, \( v_{\text{NMO}} \), and horizontal velocity \( v_H \), plotted as a function of azimuth \( \phi \) obtained from equations (15) and (16) by fitting equation (12) to the data plotted in figure 3. The normal to the fractures (\( \phi = 30^\circ \)) is easily picked from either \( v_{\text{NMO}} \) or \( v_H \). Figure 5 shows the recovered group velocity plotted as a function of polar angle for propagation at azimuths 0°, 45° and 90° with respect to the fracture normal.

Finally, a stereographic projection of the recovered group velocity is shown in figure 6. Vertical propagation corresponds to the centre of the figure while horizontal propagation corresponds to the outer edge. The orientation of the fracture set is clearly reproduced.

CONCLUSION

A non-hyperbolic moveout equation, based on an expansion of the inverse-squared group velocity in spherical harmonics, which can be used in the presence of azimuthally anisotropic layered media has been presented. This method was applied to the case of horizontal layers having monoclinic symmetry with symmetry plane parallel to the layers. A sedimentary rock containing several sets of fractures with normals lying in the bedding plane is an example of such a medium if, in the absence of fractures, the rock is transversely isotropic with symmetry axis perpendicular to the bedding plane. It was shown how the strike of the fractures can be recovered given travel time data at sufficiently many azimuths.

REFERENCES