Seismic traveltime analysis for azimuthally anisotropic media

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Summary
Natural fractures in reservoirs and in the caprock overlying the reservoir play an important role in determining fluid flow during production. Rocks possessing an anisotropic fabric and a preferred orientation of fractures display both polar and azimuthal anisotropy. A non-hyperbolic traveltime equation, suitable for azimuthally anisotropic layered media, can be obtained from an expansion of the inverse-squared ray velocity in spherical harmonics. Application of this equation to traveltime data acquired at a sufficient number of azimuths allows the principal axes of the medium to be estimated. Analysis of traveltimes measured in an azimuthally anisotropic scale-model shows the medium to be orthotropic with principal axes in agreement with those given by an independent shear-wave experiment.

Introduction
Many sedimentary rocks may be described, to a good approximation, as transversely isotropic (TI) with the symmetry axis oriented perpendicular to the bedding plane. Although reflection traveltimes have been extensively studied for TI media (Hake et al., 1984; Byun et al., 1989; Sena, 1989; Dellinger et al., 1993; Tsvankin and Thomsen, 1994; Sayers, 1995a), many formations contain fractures with orientations determined by the stress history of the rock rather than by the orientation of the bedding plane. In the case of a rock possessing both an anisotropic fabric and a preferred orientation of vertical fractures, the rock will display an azimuthal anisotropy. In this paper a non-hyperbolic traveltime equation (Sayers, 1995a,b), which can be used for azimuthally anisotropic layered media, is used to analyze ray-traced traveltimes for a fractured shale and measured traveltimes for an azimuthally anisotropic, physical model. Tsvankin and Thomsen (1994) have shown that the inversion of surface seismic data for depth-dependent anisotropic velocity models suffers from nonuniqueness because of the tradeoff between vertical velocity and depth, and the limited range of offsets normally used. In this paper use is made of wide-offset data for known receiver depths, as may be measured using multioffset/multiazimuth vertical seismic profile (VSP) surveys (Miller et al., 1994; Leaney et al., 1996).

Seismic traveltimes in azimuthally anisotropic media
It is assumed that the medium is monoclinic with a horizontal plane of mirror symmetry. A sedimentary rock containing several sets of fractures with normals lying in the bedding plane is an example of a medium with a single plane of mirror symmetry if, in the absence of fractures, the rock is transversely isotropic with the symmetry axis perpendicular to the bedding plane.

For such media, a nonhyperbolic moveout equation may be obtained using an expansion of the inverse-squared ray velocity in spherical harmonics (Sayers, 1995a,b)

\[ t^2(x) = t^2(0) + \frac{x^2}{v_{NMO}^2} - \frac{A}{(x^2 + z^2)} \]

(1)

This has the same form as that derived by Byun et al. (1989) and Sena (1989). \( t(0) \) is the vertical one-way traveltime and \( A \) is a measure of the anellipticity of the medium. \( V_{NMO} \) and \( A \) are functions of azimuth \( \phi \) and may be written in the form

\[ v_{NMO}^2 = a_1 + a_2 \cos 2\phi + a_3 \sin 2\phi \]

\[ A = a_4 + a_5 \cos 2\phi + a_6 \sin 2\phi + a_7 \cos 4\phi + a_8 \sin 4\phi. \]

In the absence of depth information, \( z^2 \) in equation (1) may be approximated by \( z^2 \approx t^2(0)/a_1 \). The horizontal velocity \( v_H \) may be estimated from \( V_{NMO} \) and \( A \):

\[ v_H = \frac{V_{NMO}}{\sqrt{1 - Av_{NMO}^2}}. \]

The coefficients \( a_i \) are linear combinations of the expansion coefficients of the inverse-squared ray velocity in spherical harmonics (Sayers, 1995b). These coefficients may therefore be obtained by fitting equation (1) to traveltime data acquired at a sufficient number of azimuths. This is illustrated below using ray-traced traveltimes for a fractured shale and measured traveltimes for an azimuthally anisotropic, physical model.

Fractured shale
To illustrate the use of equation (1), consider a horizontal shale layer with vertical axis of symmetry, \( x_3 \), containing a set of vertical fractures, with normals at \( 30^\circ \) to \( x_1 \). In the absence of fractures the shale is assumed to be TI, with elastic stiffnesses \( c_{11} = 19.19 \), \( c_{33} = 15.65 \), \( c_{44} = 7.06 \), \( c_{55} = 4.11 \), \( c_{66} = 5.70 \) GPa and density \( \rho = 2.5 \) g/cm³. The fracture set is described by parameters \( Z_N \) and \( Z_T \) (Schoenberg and Sayers, 1995). Figure 1 shows squared traveltime versus squared offset computed for \( Z_N = Z_T = 1 \), and \( 6N \cdot Z_N c_{11}^2/(1 + Z_N c_{11}^2) = 0.2 \) for a receiver at a depth of 1 km and a maximum source/receiver offset equal to twice the receiver depth. Five lines equally spaced in azimuth were used. None of these lines is parallel to the symmetry directions of the medium, the objective being to determine these directions from the data. The traveltime is nonhyperbolic as a result of the polar anisotropy of the shale.
Anisotropic traveltime analysis

(Tsvankin and Thomsen, 1995) for the $\phi = 0^\circ$ acquisition line for various $(v_{NMO}, v_H)$ pairs. Here $N$ is the number of source positions and $\Delta t_i$ is the difference between the actual traveltime and that calculated using equation (1). The resolution in $v_H$ is comparable to that in $v_{NMO}$ for the range of offsets used. The value of $\Delta t_{rms}$ corresponding to the minimum in this figure is 0.12 ms. Equation (1) therefore gives an excellent fit to the data.

Fig 1: Squared traveltime versus squared offset for a normals at $30^\circ$ to $Ox_1$. The thickness of the layer is 1 km.

A stereographic projection of the recovered ray velocity is shown in Figure 2. Vertical propagation corresponds to the center of the figure whereas horizontal propagation corresponds to the outer edge. The orientation of the fracture set, denoted by unit normal vector $n$, is clearly reproduced.

FIG. 2. Stereographic projection of the ray velocity obtained by fitting the traveltime data to equation (1).

Figure 3 shows the root-mean-square (rms) error in the traveltime:

$$\Delta t_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \Delta t_i^2},$$

where $\Delta t_i$ is the difference between the actual traveltime and that calculated using equation (1). The value of $\Delta t_{rms}$ corresponding to the minimum in this figure is 0.12 ms. Equation (1) therefore gives an excellent fit to the data.

FIG. 3. P-wave rms traveltime residuals (in ms) calculated with respect to the exact values. The maximum source/receiver offset is 2 km and the receiver depth is 1 km. The zero-offset traveltime is 0.4065 s.

Physical model

A physical model (scale-model) simulation of a reverse vertical seismic profile (RVSP) in an anisotropic medium was performed at the Allied Geophysical Laboratory, University of Houston. Physical model experiments to test seismic processing concepts are well established (see, for example, Ebrom and McDonald (1994) for a review of prior work).

Five lines of identical offset distributions, but different azimuths, were acquired over a block of Phenolite, an industrial laminate commonly used in the manufacture of electrical power grid transformers (see Figure 4). Phenolite has been used previously in a number of physical model experiments involving wave propagation in anisotropic media. The lines were placed at azimuthal angles of $30^\circ$, $102^\circ$, $174^\circ$, $246^\circ$, and $318^\circ$, measured counterclockwise from the long axis of the model.

Custom-made Harisons vertical component piezoelectric transducers with a nominal width of 3.5 mm (35 m scaled) were employed for both the source and the receiver. The source transducer was placed at the bottom of the model, simulating a down-hole source of P-waves in a borehole (such as an air gun).
Anisotropic traveltime analysis

As an example, the data from line 3 are shown in Figure 5. The P-wave direct arrival is the approximately hyperbolic event with a zero-offset arrival time of slightly more than 0.2 s (20 μs unscaled).

Plots of squared traveltime against squared offset ($t^2$ versus $x^2$) for the picked times of the direct arriving P-wave for the five lines are shown in Figures 6 and 7. Also shown is a fit of equation (1) to the picked traveltimes. The $t^2$ versus $x^2$ curves shown are concave downwards, in agreement with the expected behavior of layered sedimentary rocks (Sayers, 1995a). The azimuthal variation in traveltimes is clearly visible.
Anisotropic traveltime analysis

Application of the moveout equation to ray-traced traveltimes for a fractured shale allowed the strike of the fractures to be recovered. Use of the equation to invert measured traveltimes from a physical model simulation of a RVSP acquired over a homogeneous anisotropic medium showed the medium to be orthotropic. The orientation of the principal axes obtained was in agreement with that given by an independent shear-wave experiment using cross-polarized transducers. In contrast to previous work, no knowledge of the orientation of the symmetry planes is required. The method is therefore applicable to P-wave data collected at multiple azimuths using multiple-offset VSP techniques (Miller et al., 1994; Leaney et al., 1996).

References


FIG. 8. Stereographic projection of the ray velocity obtained by fitting the data to equation (1).

Figure 8 shows a stereographic projection of the ray velocity derived from the analysis. This is seen to possess almost perfect orthotropic symmetry, with symmetry axes at 30° and 120° clockwise rotation (seen from above) from the azimuth of line 1 (see Figure 4). These symmetry axes would also occur at rotations of 210° and 300° clockwise from line 1.

To check these results, a rotation experiment using cross-polarized shear wave transducers was performed. A shear-wave source transducer was coupled to the top surface of the test block with the polarization parallel to line 1. A shear-wave receiver, directly below the source transducer, was coupled to the lower surface of the test block with the polarization at 90° to the source transducer. The two transducers were then rotated together clockwise at 1.8° intervals, so that 200 such intervals returned the transducers to their original orientations. For an anisotropic medium, a near-zero amplitude is expected when the source transducer is aligned with one of the symmetry axes. In the experiment, extinctions were found at trace numbers 18, 68, 118 and 168. This is consistent with the extinctions expected at 30°, 120°, 210°, and 300° (see Figure 8), given the initial orientation and increment of 1.8° per trace.

Conclusion

For azimuthally anisotropic layered media, a nonhyperbolic traveltime equation can be derived using an expansion of the inverse-squared ray velocity in spherical harmonics. This approach was used for the case of horizontal layers having monoclinic symmetry with symmetry plane parallel to the layers. A sedimentary rock containing several sets of fractures with normals lying in the bedding plane is an example of such a medium if, in the absence of fractures, the rock is transversely isotropic with symmetry axis perpendicular to the bedding plane.