Seismic characterization of reservoirs containing multiple fracture sets
Colin M. Sayers*, Schlumberger

Summary

Natural fractures in reservoirs play an important role in determining fluid flow during production, and knowledge of the orientation and density of fractures is required to optimize production. Variations in reflection amplitude as a function of azimuth and incidence angle are sensitive to the presence of fractures, but current models used to invert the seismic response often make simplified assumptions that prevent fractured reservoirs from being characterized correctly. For example, many models assume a single set of perfectly aligned fractures, whereas most reservoirs contain several fracture sets with variable orientation within a given fracture set. In this paper, the variation in the reflection coefficient of seismic $P$-waves as a function of azimuth and offset is used to determine the components of a second-rank fracture compliance tensor. The variation in the trace of this tensor as a function of position in the reservoir can be used to estimate the variation in fracture density and permeability of the fracture network, and may be used to choose the location of infill wells in the field. The use of this tensor to estimate the anisotropy of the permeability tensor, the orientation of deviated wells, and the relative orientation of neighboring infill wells to ensure adequate drainage is discussed.

Introduction

Natural fractures in reservoirs play an important role in determining fluid flow during production, and knowledge of the orientation and density of fractures is required to optimize production from naturally fractured reservoirs (Reiss, 1980; Nelson, 1985). Areas of high fracture density can represent “sweet spots” of high permeability, and it is important to be able to target such locations for infill drilling. Because fractures show preferred orientations, this may result in significant permeability anisotropy in the reservoir, and it is important for optimum drainage that the separation of producers should be more closely spaced along the direction of minimum permeability than along the direction of maximum permeability.

Because oriented sets of fractures also lead to direction-dependent seismic velocities, the use of seismic waves to determine the orientation of fractures has received much attention. As an example, Lynn et al. (1994) have used the azimuthal variation in reflection amplitude of seismic $P$-waves to characterize naturally fractured gas reservoirs in the Bluebell Altamont Field in the Uinta Basin, Northeastern Utah. Reflection amplitudes offer advantages over the use of seismic velocities for characterizing fractured reservoirs because they have higher vertical resolution and are more sensitive to the properties of the reservoir. However, the interpretation of variations in reflection amplitude requires a model that allows the measured change in reflection amplitude to be inverted for the characteristics of the fractured reservoir.

Current models used to invert the seismic response of fractured reservoirs often make simplified assumptions that prevent fractured reservoirs from being characterized correctly. For example, many models assume a single set of perfectly aligned fractures (Mallick et al., 1996; Sayers and Rickett, 1997; Rueger, 2002), whereas most reservoirs contain several fracture sets with variable orientation within a given fracture set, as illustrated in Figure 1 (see, for example, Gillespie et al., 1993; Sayers, 1998; Sayers and Dean, 2001).

Use of a model of a fractured reservoir that assumes a single set of fractures in such a case can give misleading results. For example, consider a vertically fractured reservoir containing a large number of fractures with normals being isotropically distributed in the horizontal plane. For this example, there will be little or no variation in reflection coefficient with azimuth, and an interpretation of the reflection amplitude-versus-azimuth curve using an assumption of a single set of aligned fractures would predict incorrectly that the fracture density is zero.

In this paper, the variation in the reflection coefficient of seismic $P$-waves as a function of azimuth and offset is used to determine the components of a second-rank fracture compliance tensor $\alpha_{ij}$ defined by Sayers and Kachanov (1995). It is shown how the variation in the trace of this tensor as a function of position in the reservoir can be used to estimate the variation in fracture density and permeability of the fracture network, and may be used to choose the location of infill wells in the field. The use of this tensor to estimate the anisotropy of the permeability tensor, the orientation of deviated wells, and the relative orientation of neighboring infill wells to ensure adequate drainage is discussed.
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tensor as a function of position in the reservoir can be used to estimate the variation in fracture density and permeability of the fracture network. The trace of \( \alpha_{ij} \), therefore, may be used to choose the location of infill wells in the field. It is shown how the anisotropy of this tensor can be used to estimate the anisotropy of the permeability tensor, and how the principal axes of this tensor can be used to determine the orientation of deviated wells and the relative orientation of neighboring infill wells to ensure adequate drainage.

Method

Consider the reflection of seismic P-waves with angle of incidence \( \theta \) and azimuth \( \phi \) from a vertically fractured reservoir as shown schematically in Figure 2.

![Figure 2. Schematic representation of the reflection of seismic P-waves from a fractured reservoir.](image)

The axes \( x_1, x_2, x_3 \) are chosen with \( x_3 \) perpendicular to the fractured layer. In the neighborhood of the reflection point, the fractured layer is treated as an effective medium with elastic stiffness tensor \( C_{ijkl} \) and compliance tensor \( S_{ijkl} \). (Schoenberg and Sayers, 1995). These tensors vary laterally over the reservoir caused by a lateral variation in fracture density.

In the absence of fractures, the elastic stiffness tensor and elastic compliance tensor of the reservoir rock is denoted by \( C_{ijkl}^0 \) and \( S_{ijkl}^0 \), respectively. Sayers and Kachanov (1995) show that the elastic compliance of a fractured reservoir may be written in the form

\[
S_{ijkl} = S_{ijkl}^0 + \Delta S_{ijkl},
\]

where the excess compliance \( \Delta S_{ijkl} \) due to the presence of the fractures can be written as

\[
\Delta S_{ijkl} = \frac{1}{4} \left[ \delta_{il} \delta_{jk} \sigma_{ij} + \delta_{jl} \delta_{ik} \sigma_{ij} + \delta_{il} \delta_{jk} \sigma_{ij} + \delta_{jl} \delta_{ik} \sigma_{ij} \right] + \beta_{ijkl}.
\]

Here, \( \alpha_{ij} \) is a second-rank tensor, and \( \beta_{ijkl} \) is a fourth-rank tensor defined by

\[
\alpha_{ij} = \frac{1}{V} \sum_{r} B_{N}^{(r)} b_{ij}^{(r)} n_{i}^{(r)} A^{(r)}
\]

\[
\beta_{ijkl} = \frac{1}{V} \sum_{r} \left( B_{N}^{(r)} - B_{T}^{(r)} \right) b_{ijkl}^{(r)} n_{i}^{(r)} n_{j}^{(r)} n_{k}^{(r)} n_{l}^{(r)} A^{(r)}.
\]

The terms \( B_{N}^{(r)} \) and \( B_{T}^{(r)} \) are the normal and shear compliance, respectively, of the \( r \)th fracture in volume \( V \) (see Figure 3), \( n_{i}^{(r)} \) is the \( i \)th component of the normal to the \( r \)th fracture, and \( A^{(r)} \) is the area of the fracture. The tangential compliance \( B_{T} \) is assumed to be independent of the direction of the shear traction that occurs within the plane of the contact.

![Figure 3. The \( r \)th fracture in volume \( V \) has a normal \( n^{(r)} \) and surface \( A^{(r)} \).](image)

It is assumed that in the absence of fractures the reservoir is isotropic. For vertical fractures, the elastic symmetry of the fractured rock is then monoclinic, with the following nonvanishing elastic stiffness coefficients, \( C_{11}, C_{22}, C_{33}, C_{12} = C_{21}, C_{13} = C_{31}, C_{23} = C_{32}, C_{44}, C_{55}, C_{66}, C_{45} = C_{54}, C_{46} = C_{64}, C_{56} = C_{65} \), and \( C_{45} = C_{54} \) in the conventional two-index notation. For vertical fractures with normals in the \( x_1x_2 \) plane, the nonvanishing components of the excess compliance \( \Delta S_{ijkl} \) due to the presence of the fractures are (in the conventional two-index notation)

\[
\Delta S_{11} = \alpha_{11} + \beta_{1111}, \quad \Delta S_{22} = \alpha_{22} + \beta_{2222}, \quad \Delta S_{12} = \Delta S_{21} = \beta_{1122}.
\]

\[
\Delta S_{44} = \alpha_{44}, \quad \Delta S_{55} = \alpha_{55}, \quad \Delta S_{66} = (\alpha_{11} + \alpha_{22}) + 4 \beta_{1122}.
\]

\[
\Delta S_{45} = \alpha_{45} = \alpha_{54}, \quad \Delta S_{16} = \alpha_{16} + 2 \beta_{1112}, \quad \text{and} \quad \Delta S_{26} = \alpha_{26} + 2 \beta_{2222}.
\]
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The stiffness tensor of the fractured medium can then be determined by inverting the compliance tensor given by equation 1. This allows the reflection coefficient to be computed for arbitrary fracture density and contrast across the interface using, for example, the method of Schoenberg and Protazio (1992). This requires knowledge of $\alpha_1$, $\alpha_2$, $\alpha_3$, $\beta_{1111}$, $\beta_{1112}$, $\beta_{1222}$, and $\beta_{2222}$. However, if the normal and shear compliance of the fractures are approximately equal, it follows from equation 4 that the fourth-rank tensor $\beta_{ijkl}$ is small and $\Delta S_{ijkl}$ is determined by the second-rank tensor $\alpha_i$. This is a good approximation for sandstones (Sayers, 2002). With this simplification, measurements of the reflection coefficient at various offsets and azimuths can be used to determine $\alpha_1$, $\alpha_2$, and $\alpha_3$ by varying these quantities numerically to minimize $\chi^2$ defined by

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{R_{PP}^{\text{pred}}(\theta_i, \phi_i) - R_{PP}^{\text{meas}}(\theta_i, \phi_i)}{\sigma_i} \right)^2. \quad (5)$$

The term $R_{PP}^{\text{meas}}(\theta_i, \phi_i)$ is the measured PP-reflection coefficient at angle of incidence $\theta_i$ and azimuth $\phi_i$; $R_{PP}^{\text{pred}}(\theta_i, \phi_i)$ is the reflection coefficient given by theory for a given choice of $\alpha_1$, $\alpha_2$, and $\alpha_3$; $\sigma_i$ is the estimated error for measurement $i$; and $N$ is the total number of angles $\theta$ and azimuths $\phi$ pairs for which estimates of the reflection coefficient have been made. To perform the inversion, it is necessary to know the properties of the reservoir and of the overlying layer in the absence of fractures in the reservoir. These can be determined by either using measurements of reflection coefficient as a function of incidence angle and azimuth at a location where the fracture density is known to be small, or by using sonic and density logs at a well within the field to determine the properties of the matrix rock and, then, by upscaling the results to seismic wavelengths.

In many cases, the anisotropy and contrast between the overburden and reservoir can be assumed to be small. In this situation, the P-wave reflection coefficient for arbitrary elastic symmetry can be written in the form (Psenick and Vavrycuk, 1998):

$$R_{PP}(\theta, \phi) = R_{PP}^{\text{iso}}(\theta)$$

$$+ \frac{1}{2} [\Delta \delta_{11} \cos^2 \phi + (\Delta \delta_{11} - 8 \beta_{11} \Delta \gamma / \beta_{11} \beta^{11}) \sin^2 \phi + 2(\Delta \chi_{11} - 4 \beta_{11} \Delta \chi_{11} / \beta_{11} \beta^{11}) \cos \phi \sin \phi \sin^2 \theta + 2(\Delta \epsilon_{11} \cos^2 \phi + \Delta \epsilon_{11} \sin^4 \phi + \Delta \delta_{11} \cos^2 \phi \sin^2 \phi + 2(\Delta \epsilon_{11} \cos^2 \phi + \Delta \epsilon_{11} \sin^2 \phi) \cos \phi \sin \phi \sin^2 \theta \tan^2 \theta]. \quad (6)$$

where $R_{PP}^{\text{iso}}(\theta)$ denotes the weak-contrast reflection coefficient at an interface separating two slightly different isotropic media, and the anisotropy parameters $\delta_1$, $\delta_2$, $\delta_3$, $\gamma$, $\chi_1$, $\epsilon_1$, $\epsilon_2$, $\epsilon_16$, and $\epsilon_{26}$ are given by

$$\delta_x = \frac{c_{11} + 2c_{55} - c_{33}}{c_{33}}, \quad \delta_y = \frac{c_{23} + 2c_{44} - c_{33}}{c_{33}}.$$

$$\delta_z = \frac{c_{12} + 2c_{66} - c_{33}}{c_{33}}, \quad \gamma = \frac{c_{46} - c_{55}}{2c_{55}}, \quad \chi_2 = \frac{c_{26} + 2c_{45}}{c_{33}}.$$

$$\epsilon_{11} = \frac{c_{11} - c_{33}}{2c_{33}}, \quad \epsilon_y = \frac{c_{22} - c_{33}}{2c_{33}}, \quad \epsilon_16 = \frac{c_{16}}{c_{33}}, \quad \epsilon_{26} = \frac{c_{26}}{c_{33}}.$$

The elastic stiffness tensor can be found by inverting the compliance tensor given by equations 1 through 4. For small anisotropy, the elastic stiffness tensor $C$ of the fractured medium can be written in terms of the elastic stiffness tensor of the isotropic reservoir rock $C^{(0)}$ and the additional compliance $\Delta S$ due to the fractures as

$$C = C^{(0)} - C^{(0)} \Delta S C^{(0)}. \quad (7)$$

Neglecting the contribution of the fourth-rank tensor $\beta_{ijkl}$, we find that the anisotropy parameters $\delta_1$, $\delta_2$, $\delta_3$, $\gamma$, $\chi_2$, $\epsilon_1$, $\epsilon_2$, $\epsilon_16$, and $\epsilon_{26}$ are given by

$$\epsilon_1 = \frac{c_{11} + 2c_{55} - c_{33}}{c_{33}}, \quad \epsilon_2 = \frac{c_{23} + 2c_{44} - c_{33}}{c_{33}}.$$

$$\epsilon_{16} = \frac{c_{16}}{c_{33}}, \quad \epsilon_{26} = \frac{c_{26}}{c_{33}}.$$

Here, $\lambda$ and $\mu$ are the second-order elastic constants of the reservoir rock in the absence of fractures. Substitution of equation 7 into equation 5 allows $\alpha_1$, $\alpha_2$, and $\alpha_3$ to be determined using a least-squares fit to the measured PP-reflection coefficient $R_{PP}^{\text{meas}}(\theta, \phi)$ as a function of the angle of incidence $\theta$ and azimuth $\phi$.

The second-rank tensor $\alpha_i$ may be diagonalized by rotating through an angle $\theta_0$, about the axis $\chi_3$, given by

$$\tan 2\theta_0 = \frac{2\alpha_{12}}{(\alpha_{11} - \alpha_{22})}. \quad (8)$$

This angle may also be determined from the polarization of the fast-shear wave measured in a well (Sayers, 1998). Borehole seismic information, therefore, may be used to help in the inversion of reflection amplitudes for $\alpha_1$, $\alpha_2$, and $\alpha_3$, particularly in cases of poor data quality.
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Because \( \alpha_0 \) is determined by the most compliant fractures, it is expected to be a useful estimate of the orientation of the maximum permeability direction and, therefore, may be used to choose the trajectory of deviated wells designed to intersect as many open fractures as possible and to determine the relative orientation of infill wells.

Since \( \alpha_0 \) results from the presence of fractures in the reservoir it can be used to estimate the additional permeability resulting from the fractures. To do so requires knowledge of the permeability of the rock matrix, as can be determined from measurements on cores, and from estimates of the permeability tensor determined either from well tests or from reservoir history matching. In many cases, the permeability anisotropy is unknown, and only an estimate of the trace of the permeability tensor \( k_0 \) is available. A crossplot of the trace of \( \alpha_0 \) versus the trace of the permeability tensor \( k_0 \), or versus an estimate of the permeability from history matching, or from well tests allows an empirical relation to be derived that may be used to predict permeability elsewhere in the reservoir using \( \alpha_0 \) determined from an inversion of the seismic reflection amplitudes. If the anisotropy of the permeability tensor \( k_0 \) at one or more well locations is known from well tests or history matching, a similar relation between the components of \( \alpha_0 \) and the components of \( \alpha_0 \) can be used to estimate the permeability anisotropy away from the locations used for calibration. It should be noted that, in the general case, the principal axes of \( k_0 \) and \( \alpha_0 \) will not coincide, and examples for this case will be given in the presentation.

Defining a second-rank tensor \( \gamma \) by

\[
\gamma = \frac{1}{V} \sum_r g^{(r)} A^{(r)} \gamma_{(r)},
\]

where \( g^{(r)} \) is the hydraulic conductivity of the \( r \)th fracture in volume \( V \), and \( A^{(r)} \) is the area of the fracture, the permeability tensor in the presence of fractures can be written as \( k \sim k(\gamma) \) (Kachanov, 1980). If \( k_0 \) is the permeability tensor in the absence of fractures, the contribution of fractures to the permeability is given by \( k - k_0 \). If, in the absence of fractures, the permeability of the reservoir rock can be assumed to be isotropic with permeability tensor \( k_0 = k_0I \), where \( I \) is the unit tensor, \( k(\gamma) \) will be an isotropic function (if both the gradient in pressure and the fractures undergo any orthogonal transformation, then the flow undergoes the same orthogonal transformation). The Cayley-Hamilton theorem then implies that \( k - k_0I \) is a polynomial quadratic in \( \gamma \) with coefficients that are functions of the invariants of \( \gamma \) (Kachanov, 1980).

Linearizing in \( \gamma \), and using the fact that a set of parallel fractures does not affect the flow perpendicular to the fractures, allows the determination of \( k - k_0I \) as a function of \( \gamma \):

\[
k - k_0I = C(tr(\gamma)I - \gamma).
\]

This allows \( \gamma \) to be determined in the vicinity of well locations, given estimates of the permeability tensor \( k \) obtained from well tests or from reservoir history matching, and knowledge of the permeability tensor \( k_0 \) of the rock matrix. \( k_0 \) can be determined from measurements on cores.

A crossplot of the components \( \gamma_0 \) of \( \gamma \) versus the components \( \alpha_0 \) of \( \alpha \), defined by equation 3, or a crossplot of the trace of \( \gamma_0 \) versus the trace of \( \alpha_0 \) allows an empirical relation to be determined between \( \gamma_0 \) and \( \alpha_0 \) that can be used to predict permeability by using values of \( \alpha_0 \) determined from the inversion of seismic reflection amplitude away from the locations used for the calibration.

In comparing equations 3 and 9, note that \( \gamma_0 \) will be exactly proportional to \( \alpha_0 \) if either the properties of all fractures are equal or if there is a single dominant set of aligned fractures. The latter may occur in the presence of multiple sets of fractures if there is significant in-situ stress anisotropy, in which case, only the fractures with normals parallel to the minimum compressive stress are expected to have a significant effect on elastic wave propagation and fluid flow. In other cases, the proportionality between \( \gamma_0 \) and \( \alpha_0 \) will not be exact, but it is expected to lead to a useful estimate of high fracture density and, hence, higher permeability.

Conclusion

The variation in the reflection coefficient of seismic P-waves as a function of azimuth and offset can be used to determine the components of a second-rank fracture compliance tensor. The variation in the trace of this tensor as a function of position in the reservoir can be used as an estimator of the variation in fracture density and permeability of the fracture network and, therefore, may be used to select the location of infill wells in the field. Because fractures show preferred orientations, this tensor may also be used to estimate the anisotropy of the permeability tensor, the orientation of deviated wells, and the relative orientation of neighboring infill wells to ensure adequate drainage. The results are based on the assumption that the most compliant fractures have the largest hydraulic conductivity, and this assumption may be violated if compliant material, such as clay, blocks the fractures.
EDITED REFERENCES
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REFERENCES
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