Tidal-driven constraints for time-lapse reservoir monitoring
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Summary

Time-lapse seismic measurements can be used to monitor changes in saturation and pressure due to production. The interpretation of such measurements, however, requires knowledge of the variation of pore volume compressibility and fluid bulk modulus with saturation and pressure. Reservoir pressure measurements often show changes resulting from variations in hydrostatic pressure at the seafloor due to tidal effects. These variations can be analyzed to obtain constraints on the pore volume compressibility and fluid bulk modulus. With the increasing use of downhole pressure gauges, tidal measurements used together with time-lapse seismic data promise to lead to significant improvements in reservoir monitoring. Application to a North Sea data set is discussed.

Introduction

In time-lapse seismic studies, the difference between two or more seismic data sets acquired at different times during production is used to infer changes in saturation and reservoir pressure. Time-lapse seismic data are, however, sensitive to any changes in fluid bulk modulus and reservoir compressibility occurring during production. For the case of a reservoir of porosity \( \phi \) with an anisotropic frame with compliance tensor \( c_{ijkl}^{\text{frame}} \), the compliance tensor of the rock, \( c_{ijkl}^{\text{sat}} \), containing a fluid with compressibility \( c_{\text{fluid}} \) is given by (Brown and Korringa, 1975)

\[
s_{ijkl}^{\text{sat}} = s_{ijkl}^{\text{frame}} - c_{ijkl}^{\text{frame}} - \frac{c_{ijkl}^{\text{frame}} - c_0^0}{c_{\text{frame}} - c_0^0} (s_{ijkl}^{\text{frame}} - c_0^0) \phi
\]  

where \( c_{\text{frame}} \) is the compressibility of the frame. While the frame moduli, porosity and fluid bulk modulus in equation (1) can be measured in the laboratory, uncertainties exist due to the possible presence of damage in the cores taken from the reservoir, the uncertainty in the fluid bulk modulus due to the unknown distribution of fluids (patchy vs homogeneous), and the fact that core plugs sample an infinitesimal volume fraction of the reservoir. Clearly, the interpretation of time-lapse seismic measurements would be greatly aided if independent constraints on these parameters could be obtained. The purpose of this paper is to propose the use of tidal measurements for this purpose.

Rock compressibility coefficients

Consider a porous rock with bulk volume \( V_b \) and pore volume \( V_p \). Changes in \( V_b \) and \( V_p \) may occur due to changes in the confining pressure, \( P_c \), and pore pressure, \( P_p \), and may be written in terms of the four rock compressibilities \( C_{bc}, C_{bp}, C_{pc} \) and \( C_{pp} \) defined by Zimmerman (1991):

\[
C_{bc} = \frac{1}{V_p} \left( \frac{\partial V_p}{\partial P_p} \right)_{P_c}
\]  

(2)

\[
C_{bp} = \frac{1}{V_b} \left( \frac{\partial V_b}{\partial P_p} \right)_{P_c}
\]  

(3)

\[
C_{pc} = \frac{1}{V_p} \left( \frac{\partial V_p}{\partial P_c} \right)_{P_p}
\]  

(4)

\[
C_{pp} = \frac{1}{V_p} \left( \frac{\partial V_p}{\partial P_c} \right)_{P_p}
\]  

(5)

\( C_{bc} \) describes the variation in bulk volume with change in confining stress at constant pore pressure and therefore represents the compressibility of the frame. \( C_{bp} \) describes the variation in bulk volume with change in pore pressure at constant confining stress and therefore describes subsidence. \( C_{pc} \) describes the variation in pore volume with change in confining stress at constant pore pressure. \( C_{pp} \) describes the variation in pore volume with change in pore pressure at constant confining stress and is the pore volume compressibility.

Assuming the mineral grains to be microscopically homogeneous, Zimmerman (1991) shows the four rock compressibility coefficients are related by

\[
C_{bc} = C_{bp} + C_0
\]  

(6)

\[
C_{pc} = C_{pp} + C_0
\]  

(7)

\[
C_{bp} = \phi C_{pc}
\]  

(8)

where \( C_0 \) is the compressibility of the rock matrix material. It follows that a measurement of one of the rock compressibilities provides a constraint on the others. Consider, for example, the relation between well testing, which involves the pore volume compressibility \( C_{pp} \) and time-lapse seismic, which involves the frame compressibility \( C_{bc} \).
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In radial coordinates, the diffusion equation relating the variation in pressure, \( p \), with time, \( t \), is

\[
\frac{\kappa}{\mu} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \phi c \frac{\partial p}{\partial t}
\]  

(9)

where \( \kappa \) is the permeability, \( \mu \) is the fluid viscosity, \( \phi \) is the total connected porosity and \( c \) is the total compressibility of the pore/fluid system. \( c \) can be written in terms of the fluid compressibility \( c_{\text{fluid}} \) and the pore space compressibility \( c_{\text{pp}} \) as

\[
c = c_{\text{fluid}} + c_{\text{pp}}
\]

(10)

It follows from the relations of Zimmerman (1991) that

\[
C_{\text{pp}} = C_{\text{pc}} - C_0
\]

(11)

\[
C_{\text{pc}} = C_{\text{bp}} / \phi
\]

(12)

\[
C_{\text{bp}} = C_{\text{bc}} + C_0
\]

(13)

Now \( C_{\text{bc}} = C_{\text{frame}} \), so the storage capacity of the rock, \( \phi c \), may be written in the form

\[
\phi c = (C_{\text{frame}} - C_0) + (C_{\text{fluid}} - C_0) \phi
\]

(14)

which is exactly the denominator in equation (1), as pointed out by Cardona (2002). Hence

\[
S_{ijkl} = S_{ijkl}^{\text{frame}} - (S_{ijkl}^{\text{frame}} - S_{ijkl}^{\text{0}})(S_{ijkl}^{\text{frame}} - S_{ijkl}^{\text{0}}) / \phi c
\]

(15)

where \( \phi c \) may be constrained using well test information.

Changes in reservoir pressure caused by tides

Changes in sea level due to tidal variations result in changes in reservoir pressure as illustrated in figure 1. Variations in reservoir pressure due to tidal variations have been observed in several North Sea oilfields (Dean et al., 1991). Earlier evidence for variations in aquifer pressure due to ocean tides comes from measurements of water levels in wells (Wang, 2000). As early as 77 A.D., Pliny the Elder reported variations in water level in a well near the temple of Hercules in Cadiz related to ocean tides (Melchior, 1983; Wang, 2000). In 1902 the United States Geological Society reported water-level variations in wells in Atlantic City, New Jersey that varied with ocean depth (Meinzer, 1928; Wang, 2000).

For simplicity, consider the case of an isotropic reservoir. The equations of poroelasticity are then:

\[
\epsilon_1 = 1 \frac{\Delta \sigma_1 - \frac{3\nu}{(1+\nu)} \Delta \sigma_m}{3} - \frac{C_m}{3} \Delta P_p
\]

(16)

\[
\epsilon_2 = 1 \frac{\Delta \sigma_2 - \frac{3\nu}{(1+\nu)} \Delta \sigma_m}{3} - \frac{C_m}{3} \Delta P_p
\]

(17)

\[
\epsilon_3 = 1 \frac{\Delta \sigma_3 - \frac{3\nu}{(1+\nu)} \Delta \sigma_m}{3} - \frac{C_m}{3} \Delta P_p
\]

(18)

where \( \Delta \sigma_m = (\Delta \sigma_{11} + \Delta \sigma_{22} + \Delta \sigma_{33}) / 3 \). Defining \( \epsilon_m = (\epsilon_1 + \epsilon_2 + \epsilon_3) / 3 \), it follows that

\[
\Delta \sigma_{11} = 2\epsilon_1 + \frac{3\lambda \epsilon_m + \alpha \Delta P_p}{\epsilon_m}
\]

(19)

\[
\Delta \sigma_{22} = 2\epsilon_2 + \frac{3\lambda \epsilon_m + \alpha \Delta P_p}{\epsilon_m}
\]

(20)

\[
\Delta \sigma_{33} = 2\epsilon_3 + \frac{3\lambda \epsilon_m + \alpha \Delta P_p}{\epsilon_m}
\]

(21)

where

\[
\alpha = \frac{C_0}{C_{\text{bc}}} = 1 - \frac{K}{K_0}
\]

(22)

Hence

\[
\Delta P_p = \frac{(1 + \nu)}{3(1 - \nu)} \frac{C_{\text{pc}}}{\epsilon_m}
\]

(23)

It should be noted that equation (23) is valid for any Biot constant, and therefore can be applied for both loading and unloading as shown in figure 2.
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Figure 2. $\alpha = 1 - K / K_0$ may be significantly less than 1 for unloading (e.g. water injection).

Now from Zimmerman (2002),

\[ C_{\text{uniaxial}} = C_{pp} - \frac{2(1 - 2\nu)}{3(1 - \nu)} \alpha C_{pc} \]  
(24)

\[ C_{\text{uniaxial}} = \frac{(1 + \nu)}{3(1 - \nu)} C_{pc} \]  
(25)

Hence

\[ \frac{\Delta P_p}{\Delta \sigma_z} = \frac{C_{\text{uniaxial}}}{C_{pc}} = \frac{2(1 - 2\nu)}{3(1 - \nu)} \frac{\epsilon_r}{\phi \Delta P_p} + C_{\text{fluid}} \]  
(26)

In the limit of incompressible grains

\[ C_{pc} = C_{pp} + C_0 \rightarrow C_{pp}, \quad \alpha = 1 - \frac{K}{K_0} \rightarrow 1, \]

\[ \Rightarrow C_{\text{uniaxial}} = C_{pp} - \frac{2(1 - 2\nu)}{3(1 - \nu)} \alpha C_{pc} \rightarrow \frac{(1 + \nu)}{3(1 - \nu)} C_{pp} \]

\[ \Rightarrow C_{\text{uniaxial}} = \frac{(1 + \nu)}{3(1 - \nu)} C_{pc} \rightarrow C_{\text{uniaxial}}. \]

Hence

\[ \frac{\Delta P_p}{\Delta \sigma_z} \rightarrow \frac{C_{\text{uniaxial}}}{C_{pp}} = \frac{2(1 - 2\nu)}{3(1 - \nu)} \frac{\epsilon_r}{\phi \Delta P_p} + C_{\text{fluid}} \]  
(27)

Assuming zero lateral strain, $\epsilon_r = 0$,

\[ \frac{\Delta P_p}{\Delta \sigma_z} \rightarrow \frac{1}{1 + C_{\text{fluid}} / C_{pp}} \]  
(28)

North Sea example

Figure 3 shows an example of a tidal measurement from the North Sea. The tide predicted for this location is shown in figure 4, and a comparison of the predicted variation in reservoir pressure with the measured reservoir pressure is compared in figure 5. The agreement is seen to be excellent. Application of the theory then allows the pore volume compressibility of the rock to be determined.

Figure 3. An example of a tidal measurement from the North Sea.

Figure 4. The tide predicted for the location of the North Sea example.
Conclusion

Although time-lapse seismic measurements have been used widely to monitor changes in saturation and pressure due to production, reliable estimates of the change in saturation and fluid pressure in the reservoir require knowledge of the variation in pore volume compressibility and fluid bulk modulus with fluid saturation and reservoir pressure. These variations can be studied in the laboratory, but uncertainties exist due to the possible presence of damage in rock samples resulting from the stress release that occurs during coring, the uncertainty in the fluid bulk modulus due to the unknown distribution of fluids (patchy vs homogeneous), and the fact that core plugs sample an infinitesimal volume fraction of the reservoir.

Changes in sea level due to tides lead to variations in reservoir pressure that can be measured using downhole pressure sensors. These variations can be analyzed to obtain constraints on the pore volume compressibility and fluid bulk modulus, and therefore provide important constraints for the interpretation of time-lapse seismic measurements. It is expected that with the increasing use of downhole pressure sensors, measurements of changes in reservoir pressure due to tidal effects will become more routine and, in combination with time-lapse seismic measurements, will be used increasingly to monitor changes in saturation and pressure resulting from production.

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References


Meinzer, O.E., 1928, Compressibility and elasticity of artesian aquifers, Econ. Geol. 23, 263-291.


