The sensitivity of seismic waves to the normal and shear compliance of fractures

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Summary

Schoenberg (1980) proposed a model for an imperfectly bonded interface between two elastic media in which the displacement discontinuity is taken to be linearly related to the stress traction that is continuous at the interface. For isotropic interface behavior, there are two interface compliances $B_N$ and $B_T$, where the displacement discontinuity normal to the interface is given by $B_N$ times the normal stress, and the displacement discontinuity tangential to the interface is given by $B_T$ times the shear stress and acts in the same direction. This model has been widely used to describe the seismic response of fractured reservoirs. Simple asperity deformation models of a fracture suggest that $B_N / B_T = 1$ for dry fractures in a rock with low Poisson's ratio. However, more realistic models, which take into account the deformation of the void space between the fracture faces, lead to significant departures of $B_N / B_T$ from unity, and the implications of this for the seismic characterization of fractured reservoirs are discussed.

Introduction

Naturally occurring fractures often act as the primary flow paths in low-permeability rocks as well as shortcuts for flow in higher-permeability formations. Because natural fractures show preferred orientations, they may lead to significant permeability anisotropy in the reservoir. In choosing the location of infill wells, it is important for optimal drainage that the producer wells should be more closely spaced along the direction of minimum permeability than along the direction of maximum permeability. As a result, it is desirable to be able to characterize the orientation and density of fractures in order to optimize production from naturally fractured reservoirs. Because of this, the use of seismic waves to determine the orientation of fractures has received much attention.

A fracture can be modeled as an imperfectly bonded interface, across which the traction $t$ is continuous, but the displacement $u$ may be discontinuous (Schoenberg, 1980). It is convenient to denote the discontinuity in displacement across the interface by $[u]$. Assuming that the displacement discontinuity $[u]$ is linear in the traction, the $i$th component of the displacement discontinuity may be written as

$$u_i = B_{ij}t_j,$$  \hspace{1cm} (1)

where $t_j$ is the $j$th component of the traction vector, and $B_{ij}$ is the fracture compliance matrix. If there is rotational symmetry around the normal to the fracture it follows that $B_{ij}$ may be represented in terms of a normal compliance $B_N$ and shear compliance $B_T$ as follows:

$$B_{ij} = B_N n_i n_j + B_T \{ \delta_{ij} - n_i n_j \},$$  \hspace{1cm} (2)

where $n_i$ is the $i$th component of the normal $n$ to the fracture. Choosing a reference set of axes $x_1,x_2,x_3$ with $x_3$ normal to the fracture, equation (2) may be written as

$$[u_3] = B_N t_3, \quad [u_{1\alpha}] = B_T t_{1\alpha}, \quad \text{for } \alpha = 1 \text{ or } 2.$$ \hspace{1cm} (3)

Thus $B_N$ gives the displacement discontinuity normal to the fracture for unit normal traction, while $B_T$ gives the displacement discontinuity parallel to the fracture for unit shear traction.

The effective elastic stiffness tensor of an isotropic background containing an arbitrary orientation distribution of fractures is orthotropic (orthorhombic) in the long-wave limit with three orthogonal planes of mirror symmetry if $B_N / B_T = 1$ for all fractures (Kachanov, 1980; Sayers and Kachanov, 1991; Schoenberg and Sayers, 1995). However, deviations from orthotropy may occur if $B_N / B_T$ differs significantly from unity. In this paper, the possibility that $B_N / B_T$ may depart significantly from unity is examined, and the implications for the seismic characterization of fractured reservoirs is discussed.

The normal and shear compliance of fractures

To examine possible values of the ratio $B_N / B_T$ of the normal and shear compliance, it is useful to consider various microstructural models of a fracture. The simplest approach is to represent the fracture as two rough surfaces in contact. White (1983) treated the properties of a fracture by assuming that each fracture plane is somewhat irregular, and that tangential movement has displaced the matching surfaces sufficiently that contact between the fracture faces only occurs at localized spots where bulges happen to meet. The deformation of the contacts was treated using the classical Hertz-Mindlin contact theory. This model gives
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\[ B_N / B_T = \frac{1 - \nu}{1 - \nu/2}, \]  

(4)

where \( \nu \) is the Poisson’s ratio of the background rock.

Despite the popularity of asperity deformation models for estimating the normal and shear compliances of fractures based on Hertz-Mindlin theory, Xu and King (1992) showed that the asperity deformation was typically two orders of magnitude lower than the deformation of the void space between the faces of the fracture. This is consistent with the model proposed by Nagy (1992), who emphasized the importance of the volumetric nature of the cavities entrapped between compressed rough surfaces.

To see the importance of the deformation of the void space between the fracture faces, consider a fracture modeled as a planar array of empty ellipsoidal voids following Baik and Thompson (1984) and Nagy (1992). The voids are assumed to be oblate spheroids with in-plane radius \( a \) and out-of-plane radius \( c \), as shown in Figure 1.

Baik and Thompson (1984) gave an expression for the normal compliance \( B_N \), while Nagy (1992) gave an expression for the shear compliance based on the solution of Eshelby (1961). Figure 2 shows the variation in \( B_N / B_T \) as a function of aspect ratio \( c/a \) for various values of Poisson’s ratio of the background medium. The fracture is assumed to be dry. For very small aspect ratios, \( B_N / B_T \) approaches the value \( B_N / B_T = 1 - \nu/2 \) corresponding to a circular crack (Sayers and Kachanov, 1995), but decreases smoothly with increasing aspect ratio.

Another form of contact between the surfaces of naturally occurring fractures arises from mineralization in the form of bridges between opposing faces of the fracture (Laubach et al. 2004, Laubach and Ward 2006). Sayers et al. (2009) examined the effect of such bridges on the ratio \( B_N / B_T \) using finite-element modeling techniques, and found that the presence of such bridges may lead to a significant decrease in \( B_N / B_T \) from unity.

Reflectance amplitudes offer advantages over seismic velocities for use in characterizing fractured reservoirs because they have higher vertical resolution and are more sensitive to the properties of the reservoir. In this section, the effect of variable \( B_N / B_T \) on the variation in amplitude versus offset and azimuth (AVOA) is investigated for the case of two nonorthogonal sets of vertical fractures as illustrated in Figure 3.

For an elastic medium containing an arbitrary orientation distribution of fractures, Sayers and Kachanov (1995) show that the elastic compliance tensor may be written in the form
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\[ S_{ijkl} = S_{ijkl}^0 + \Delta S_{ijkl}, \]  

where the excess compliance \( \Delta S_{ijkl} \) that is due to the presence of the fractures can be written as

\[ \Delta S_{ijkl} = \frac{1}{V} \left[ \delta_{ij} \alpha_{kl} + \delta_{ik} \alpha_{jl} + \delta_{il} \alpha_{jk} + \delta_{jl} \alpha_{ij} \right] + \beta_{ijkl}. \]  

Here, \( \alpha_{ij} \) and \( \beta_{ijkl} \) are the second- and fourth-rank fracture compliance tensor defined by

\[ \alpha_{ij} = \frac{1}{V} \sum_{r} B_{ij}^{(r)} n_{r}^{(i)} n_{r}^{(j)} A^{(r)} \]  

and

\[ \beta_{ijkl} = \frac{1}{V} \sum_{r} (B_{ij}^{(r)} - B_{ij}^{(0)}) n_{r}^{(i)} n_{r}^{(j)} n_{r}^{(k)} n_{r}^{(l)} A^{(r)}. \]  

These tensors represent the maximum information that can be obtained about a fractured reservoir from static elastic properties alone. The terms \( B_{ij}^{(r)} \) and \( B_{ij}^{(0)} \) are the normal and shear compliances of the \( r \)th fracture in volume \( V \), \( n_{r}^{(i)} \) is the \( i \)th component of the normal to the \( r \)th fracture, and \( A^{(r)} \) is the area of the fracture.

If \( B_{ij}^{(r)} = B_{ij}^{(0)} \) for all fractures, the elastic stiffness tensor is completely determined by the second-rank tensor \( \alpha_{ij} \). The elastic stiffness tensor is then orthotropic (orthorhombic), with principal axes coinciding with the principal axes of the second-rank tensor \( \alpha_{ij} \) (Kachanov, 1980; Sayers and Kachanov, 1991; Schoenberg and Sayers, 1995). However, if \( B_{ij}^{(r)} \neq B_{ij}^{(0)} \), the fourth-rank tensor \( \beta_{ijkl} \) is nonzero and may cause deviations from orthotropy, with important consequences for seismic wave propagation.

For two sets of vertical fractures with azimuths \( \phi^{(1)} \) and \( \phi^{(2)} \), as shown in Figure 3, the tensors \( \alpha_{ij} \) and \( \beta_{ijkl} \) defined by equations (7) and (8) can be written in the form

\[ \alpha_{ij} = A_{1} n_{ij}^{(1)} n_{ij}^{(1)} + A_{2} n_{ij}^{(2)} n_{ij}^{(2)} \]  

and

\[ \beta_{ijkl} = B_{ijkl}^{(1)} n_{ij}^{(1)} n_{kl}^{(1)} n_{kl}^{(1)} n_{ij}^{(2)} n_{kl}^{(2)} n_{ij}^{(2)} n_{kl}^{(2)} \]  

(Sayers and Dean, 2001). Let \( A_{1} = (1 - \eta) A \), and \( A_{2} = \eta A \). The variation in the azimuth \( \phi_{2} \) of the fast polarization direction as a function of \( \eta \) for vertically propagating shear waves has been discussed by Sayers (1998) and is given by

\[ \tan 2\phi_{2} = 2\alpha_{12} / (\alpha_{11} - \alpha_{22}). \]  

To illustrate the effects of departures of \( B_{ij} / B_{ij} \) from unity on AVOA, the parameters given by Allen and Peddy (1993) for the case of Taylor Shale over unfractured Austin Chalk listed in Table 1 will be used. In the absence of fractures both formations are assumed to be isotropic. In this case, the critical angle for PP-reflections is 56.7°.

Table 1: Parameters given by Allen and Peddy (1993) for the case of Taylor Shale over unfractured Austin Chalk.

<table>
<thead>
<tr>
<th>Formation</th>
<th>( \rho ) [g/cm³]</th>
<th>( v_{p} ) [km/s]</th>
<th>( v_{s} ) [km/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taylor Shale</td>
<td>2.60</td>
<td>4.153</td>
<td>2.419</td>
</tr>
<tr>
<td>Austin Chalk</td>
<td>2.57</td>
<td>4.970</td>
<td>2.615</td>
</tr>
</tbody>
</table>

The PP-AVOA response for the case of Taylor Shale over Austin Chalk containing two nonorthogonal sets of fractures was calculated using the approach of Schoenberg and Protazio (1992). In this approach, the reflectivity and transmissivity matrices are derived for plane waves incident on a horizontal interface in terms of the horizontal slowness. This method is suitable for any model with a horizontal plane of reflection symmetry.

Figures 4-6 show the PP-reflection coefficient decomposed into Fourier components \( \cos 2(\phi - \phi_{i}) \) and \( \cos 4(\phi - \phi_{i}) \) for two fracture sets with azimuths \( \phi^{(1)} = 0° \), \( \phi^{(2)} = 60° \), \( \eta = 0.3 \), and angle of incidence \( \theta = 45° \) for the cases \( B_{ij} = B_{ijkl} \) (Figure 4), \( B_{ij} = B_{ijkl} / 2 \) (Figure 5) and \( B_{ij} = 0 \) (Figure 6).
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When $N_T/B_B = 1$, as in Figure 4, the principal axes of the Fourier components $\cos(2(\phi - \phi_1))$ (blue curve) and $\cos(4(\phi - \phi_1))$ (green curve) for two fracture sets with azimuths $\phi^{(1)} = 0^\circ$, $\phi^{(2)} = 60^\circ$, $B_N = B_T / 2$, $\eta = 0.3$, and angle of incidence $\theta = 45^\circ$. The average value is indicated by the horizontal line.

When $B_N = B_T$, as in Figure 4, the principal axes of the Fourier components $\cos(2(\phi - \phi_1))$ and $\cos(4(\phi - \phi_1))$ coincide ($\phi_1 = \phi_3$), and are aligned with the fast shear polarization direction of a vertically propagating shear wave. When $B_N \neq B_T$ as in Figure 5 and 6, $\phi_1$ and $\phi_3$ do not coincide. However, at angles of incidence $\theta = 45^\circ$, $\phi_1$ is seen to be close to the normal-incidence fast shear polarization azimuth $\phi_{31}$ for the values of $B_N / B_T$ considered. This is not true for the $4\phi$ component with $\phi_1$ deviating increasingly with decrease in $B_N / B_T$. The amplitude of the $\cos(4(\phi - \phi_1))$ is seen to increase with decreasing $B_N / B_T$, as expected from equation (8).

Conclusion

The displacement discontinuity model of Schoenberg (1980) is widely used to describe the seismic response of fractured reservoirs. The ratio of the normal to shear compliance $B_N / B_T$ of the fractures is an important quantity for the seismic characterization of fractured reservoirs, and it has been suggested as a fluid indicator. The effective elastic stiffness tensor of an isotropic background containing an arbitrary orientation distribution of fractures is orthotropic (orthorhombic) in the long-wave limit with three orthogonal planes of mirror symmetry if $B_N / B_T = 1$ for all fractures. However, deviations from orthotropy may occur if $B_N / B_T$ differs significantly from unity.

Simple asperity deformation models of a fracture such as that of White (1983) suggest that $B_N / B_T \approx 1$ for dry fractures in a rock with low Poisson's ratio. Despite the popularity of such models, however, Xu and King (1992) showed that the asperity deformation was typically two orders of magnitude lower than the deformation of the void space between the faces of the fracture. This is consistent with the model proposed by Nagy (1992), who emphasized the importance of the volumetric nature of the cavities entrapped between compressed rough surfaces.

A simple model of a fracture as a planar array of ellipsoidal voids following Baik and Thompson (1984) and Nagy (1992) shows that the ratio of the normal to shear compliance $B_N / B_T$ decreases as the aspect ratio of the voids between the fractures increases, and as the Poisson's ratio of the background rock increases. This can cause the maxima and minima in the variation in the $PP$-reflection amplitude as a function of azimuth to deviate from the fast and slow polarization direction of a vertically propagating $S$-wave. Caution therefore needs to be exercised in using amplitude versus azimuth data for predicting fluids in fractured reservoirs.
EDITED REFERENCES
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